EE 428 **EXAM II** **21 November 1996**

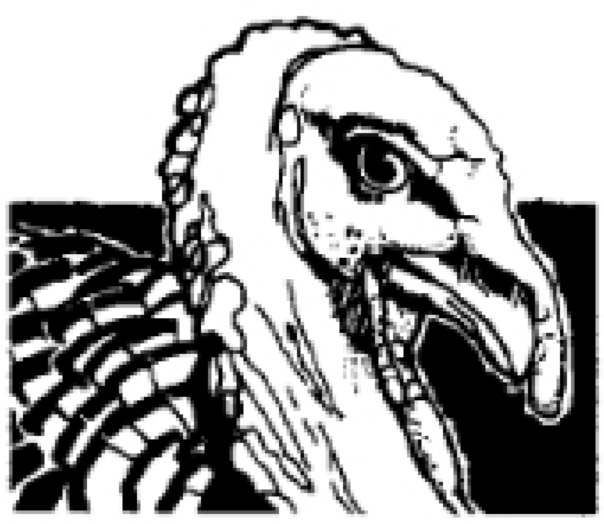
Name: Solutions

ID#: ____________________

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Problem	Weight	Score
1	30	
2	30	
3	40	
Total	100	

This test consists of three problems. Answer each problem on the exam itself; if you use additional paper, repeat the identifying information above, and staple it to the rest of your exam when you hand it in. The quality of your analysis and evaluation is as important as your answers. Your reasoning must be precise and clear; your complete English sentences should convey what you are doing.



Don't even think about it . . .

Problem 1: (30 points)

The system in Fig. 1 contains a unity-feedback loop containing a minor-rate-feedback loop.

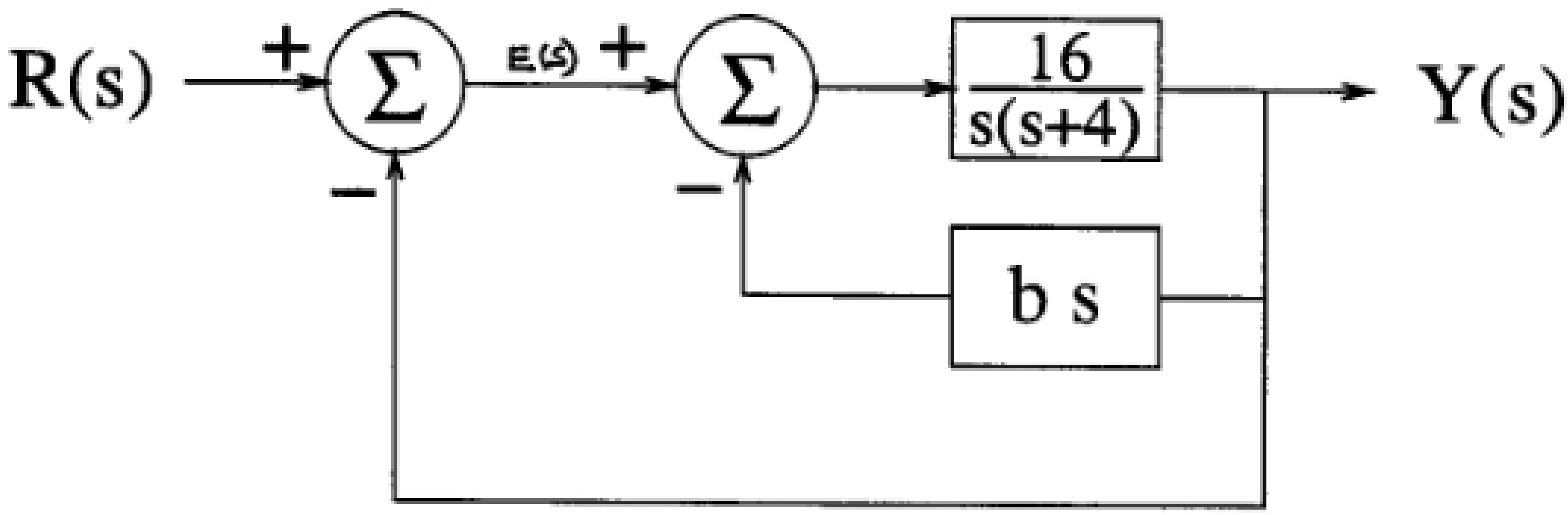


Figure 1: Minor-loop feedback compensation.

1. (3 points) Calculate the transfer function $Y(s)/R(s)$.
2. (2 points) Write the characteristic equation for the system.
3. (15 points) Sketch the root locus of the system for $b \geq 0$ on on the graph provided on page 5.
   (a) (3 points) Express the characteristic equation as
   $$1 + KG(s) = 0,$$
   where both the numerator and denominator of $G(s)$ are monic polynomials in s. Specify $G(s)$ and K.
   (b) (3 points) Calculate the asymptote(s).
   (c) (3 points) Compute the departure and arrival angles.
   (d) (6 points) If there are either breakaway or breakin points, calculate their their location.
4. (5 points) Using the root locus sketch, calculate the range of values of b for which the system is BIBO stable.
5. (5 points) Using the root locus sketch, calculate the value of b for which the transient response of the closed-loop system is critically damped.

1. The transfer function $Y(s)/E(s)$ of the inner loop is

$$\frac{Y(s)}{E(s)} = \frac{16}{s^2 + (4+16b)s}$$. Using this result

$$\boxed{\frac{Y(s)}{R(s)} = \frac{Y(s)/E(s)}{1 + Y(s)/E(s)} = \frac{16}{s^2 + (4+16b)s + 16}}$$

2. The characteristic equation of the closed-loop system is

$$\boxed{s^2 + (4+16b)s + 16 = 0}$$

3. (a) To sketch the root locus as a function of b, express the characteristic equation as

$$(s^2 + 4s + 16) + 16bs = 0$$

$$1 + \underbrace{\boxed{16b}}_{\equiv K} \underbrace{\boxed{\frac{s}{s^2 + 4s + 16}}}_{\equiv G(s)} = 0$$

(b) $n = 2$ branches, $m = 1$ finite zero, $n - m = 1$ zero at infinity.

$\gamma_\ell = \dfrac{180 + 360\ell}{n - m}$ $\boxed{\gamma_0 = 180°}$. One pole moves to the left along the real axis as $K \to \infty$.

(c) The open-loop poles of $G(s)$ are located at

$$p = -2 + j\sqrt{-4+16} = -2 + j2\sqrt{3}$$

and $p^* = -2 - j2\sqrt{3}$. The angle of departure at p is obtained by placing a test point s_t close to $-2 + j2\sqrt{3}$ so that

$$180° + 360\ell = \measuredangle G(s_t) = \measuredangle \frac{s_t}{(s_t - p)(s_t - p^*)}$$

$$= \underbrace{\underbrace{\measuredangle s_t}_{90° + \tan^{-1}\frac{2}{2\sqrt{3}}}}_{120°} - \underbrace{\measuredangle (s_t - p)}_{\phi_d} - \underbrace{\measuredangle (s_t - p^*)}_{90°}$$

$$\phi_d = -180° - 360\ell + 120° - 90° = -150° \quad \ell = 0$$

$$\boxed{\phi_d = -150°}$$