

Name: Solutions

ID#: \_\_\_\_\_

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Problem	Weight	Score
1	30	
2	30	
3	20	
4	20	
Total	100	

This test consists of four problems. Answer each problem on the exam itself; if you use additional paper, repeat the identifying information above, and staple it to the rest of your exam when you hand it in. The quality of your analysis and evaluation is as important as your answers. Your reasoning must be precise and clear; your complete English sentences should convey what you are doing.

**Problem 1: (30 points)**

Figure 1 shows the block diagram of a position control system. The plant is a DC motor with input armature voltage  $u(t)$  and output angular displacement  $\theta(t)$ . The Bode magnitude and phase response of  $\Theta(s)/U(s)$  are shown in Figure 2. In this problem you will design a cascade controller so that the closed-loop system meets the following design specifications.

- i. A velocity error constant  $K_v = 10$ .
- ii. A phase margin greater than  $40^\circ$  for the compensated open-loop transfer function  $G_c(s)G_p(s)$ .

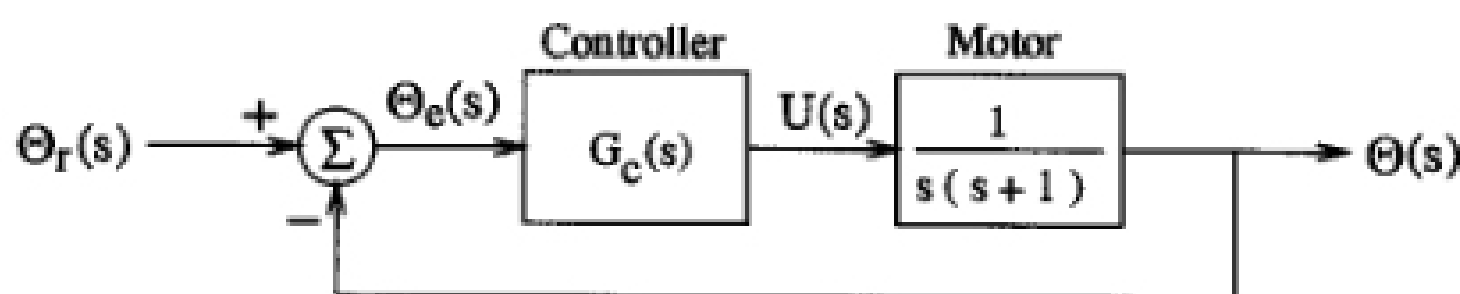


Figure 1: Position control system with cascade compensation.

1. (5 points) What is the phase margin of the uncompensated open-loop transfer function  $G_p(s)$  ?
2. (5 points) Suppose proportional control  $G_c(s) = K_o$  is used. Specify the value of  $K_o$  required so that the design specification (i) is met.
3. (5 points) Can both design specifications be met simultaneously using proportional control  $G_c(s) = K_o$  ? Justify your answer.
4. (15 points) Design a phase lead controller

$$G_c(s) = K_o \frac{\left(1 + \frac{s}{\omega_o}\right)}{\left(1 + \frac{s}{k \omega_o}\right)}$$

that satisfies both design specifications. Clearly show how the parameters  $K_o$ ,  $k$ , and  $\omega_o$  were obtained.

1. From Figure 2,  $\omega_{gc} = 0.8 \frac{\text{rad}}{\text{sec}}$ ,  $PM \approx 55^\circ$
2.  $e_{ss} = \frac{1}{K_v}$   $K_v = \lim_{s \rightarrow 0} s K_o \frac{1}{s(s+1)} = K_o \Rightarrow$   $K_o = 10$
3. If  $K_o = 10$ , the resulting PM of  $K_o G_p(s)$  is  $20^\circ$ .  
(The gain crossover freq of  $10 G_p(s)$  is  $3 \text{ rad/sec}$ )

Choosing  $K_o = 10$  to satisfy  $K_v = 10$  will result in an inadequate PM. Proportional control cannot simultaneously satisfy both design goals.

# Bode Diagrams

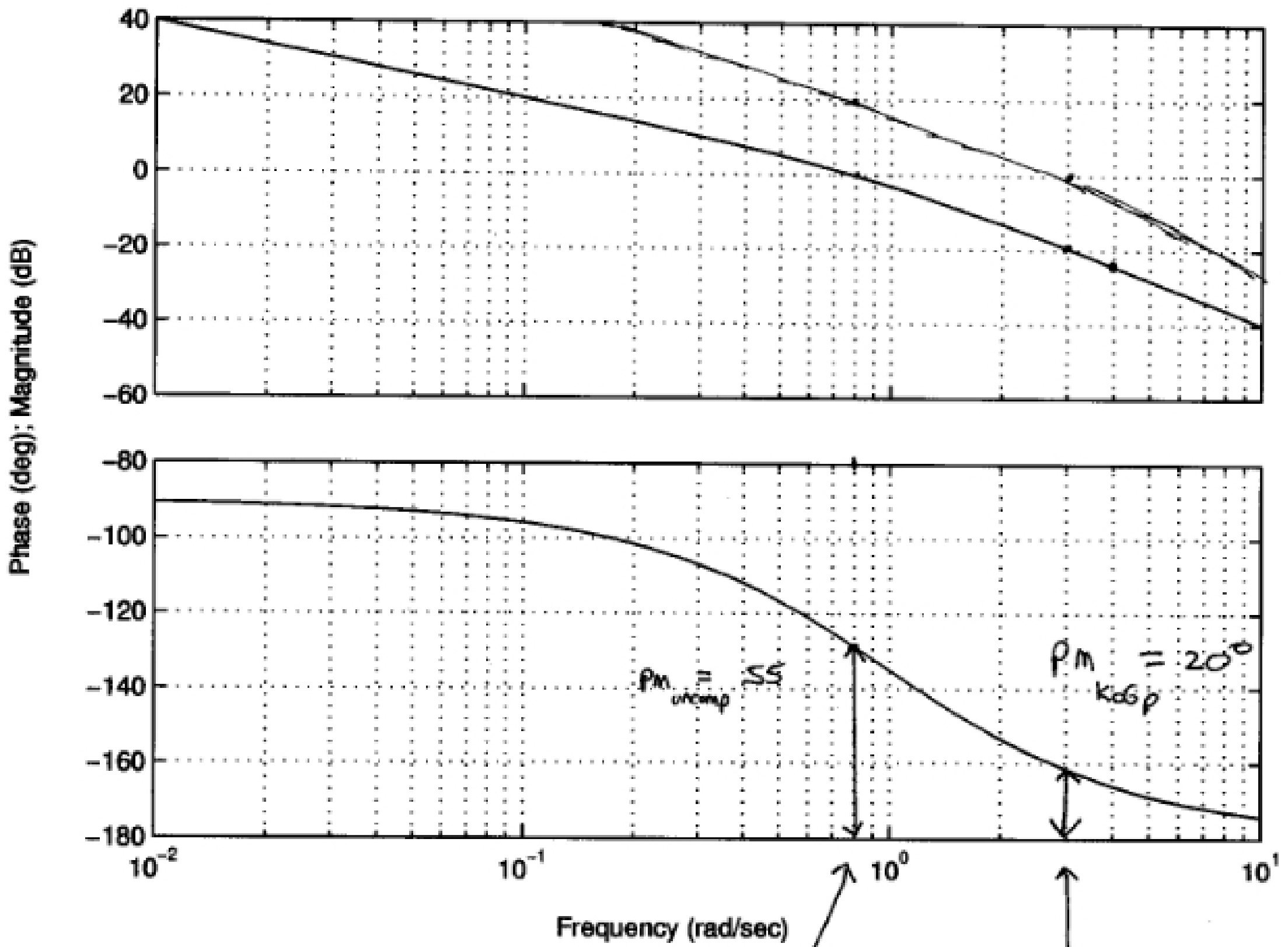


Figure 2: Bode magnitude and phase plot of  $G_p(s)$ .

gain crossover frequency of  $G_p(s)$ .

gain crossover frequency of  $10 G_p(s)$