

This exam contains ten multiple-choice problems worth two points each, five matching problems worth one point each, and four computational problems worth 25 points altogether, for an exam total of 50 points.

Part I. Multiple Choice. (2 points each)

For each of the following, mark your answer card with the letter corresponding to the only correct answer.

1. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2 + 2y^2}{xy}$. What substitution would be effective in solving this equation, and how would it help?

- (A) $v = \frac{y}{x}$ It would make the differential equation separable.
- (B) $v = \frac{x}{y}$ It would make the differential equation linear.
- (C) $v = x + y$ It would make the differential equation separable.
- (D) $v = x - y$ It would make the differential equation linear.
- (E) $y = \frac{1}{x}$ It would make the differential equation separable.
- (F) $y = \frac{1}{x}$ It would make the differential equation linear.
- (G) $y = 1 - x$ It would make the differential equation separable.
- (H) $y = 1 - x$ It would make the differential equation linear.

2. Consider the initial value problem $y'' + 5y' + 4y = 0$, $y(0) = 1$, $y'(0) = 2$. What is the behavior of the solution as $t \rightarrow \infty$?

- (A) $y \rightarrow -\infty$
- (B) $y \rightarrow -4$
- (C) $y \rightarrow -2$
- (D) $y \rightarrow -1$
- (E) $y \rightarrow 0$
- (F) $y \rightarrow 1$
- (G) $y \rightarrow 2$
- (H) $y \rightarrow 4$
- (I) $y \rightarrow \infty$

3. Consider the linear nonhomogeneous differential equation $y'' + p(t)y' + q(t)y = g(t)$ as well as the corresponding homogeneous equation $y'' + p(t)y' + q(t)y = 0$.

Suppose y_1 and y_2 are solutions of the nonhomogeneous differential equation, and y_3 and y_4 are solutions of the homogeneous differential equation.

Three of the following statements are true and two are false. Which are the two false ones?

- (I) $y_1 + y_2$ is a solution of the nonhomogeneous differential equation.
- (II) $y_3 + y_4$ is a solution of the homogeneous differential equation.
- (III) $y_1 + y_3$ is a solution of the nonhomogeneous differential equation.
- (IV) $5y_1$ is a solution of the nonhomogeneous differential equation.
- (V) $5y_3$ is a solution of the homogeneous differential equation.

- (A) I and II
- (B) I and III
- (C) I and IV
- (D) I and V
- (E) II and III
- (F) II and IV
- (G) II and V
- (H) III and IV
- (I) III and V
- (J) IV and V

4. Consider the nonhomogeneous differential equation

$$y'' - 10y' + 25y = 6e^{5t}$$

What is the correct form which must be used to find a particular solution Y when using the method of undetermined coefficients?

- (A) $Y = Ae^{5t}$
- (B) $Y = Ate^{5t}$
- (C) $Y = Ate^{5t} + Be^{5t}$
- (D) $Y = At^2e^{5t}$
- (E) $Y = At^2e^{5t} + Bte^{5t}$
- (F) $Y = At^2e^{5t} + Bte^{5t} + Ce^{5t}$
- (G) $Y = At^3e^{5t}$
- (H) $Y = At^3e^{5t} + Bt^2e^{5t}$
- (I) $Y = At^3e^{5t} + Bt^2e^{5t} + Cte^{5t}$
- (J) $Y = At^3e^{5t} + Bt^2e^{5t} + Cte^{5t} + De^{5t}$

5. Consider the differential equation $x^2y'' - 3xy' + 3y = 0$. Only one of the following is a fundamental set of solutions for this differential equation. Which one is it? (Hint: You have not yet learned how to solve this type of differential equation, so you should be able to answer the question without going through a solving procedure.)

- (A) $f_1 = x$
- (B) $f_1 = x^2$
- (C) $f_1 = x^3$
- (D) $f_1 = x, f_2 = x^2$
- (E) $f_1 = x, f_2 = x^3$
- (F) $f_1 = x^2, f_2 = x^3$
- (G) $f_1 = x, f_2 = 2x$
- (H) $f_1 = x, f_2 = x^2, f_3 = x^3$
- (I) $f_1 = x, f_2 = x^3, f_3 = x + x^3$
- (J) $f_1 = x^2, f_2 = 2x^2, f_3 = 3x^2$

6. A spring oscillates according to the equation of motion $u = 2\cos 2t - \sin 2t$, where t is measured in seconds and u is measured in feet. Find the period and the amplitude of the motion.

- (A) π seconds; 2 feet
- (B) π seconds; 3 feet
- (C) π seconds; $\sqrt{5}$ feet
- (D) 2π seconds; 2 feet
- (E) 2π seconds; 3 feet
- (F) 2π seconds; $\sqrt{5}$ feet
- (G) 4π seconds; 2 feet
- (H) 4π seconds; 3 feet
- (I) 4π seconds; $\sqrt{5}$ feet