Differential Equations (Math 217) Final Exam December 18, 2003

This exam contains twenty multiple-choice problems worth five points each for an exam total of 100 points. For each problem, mark your answer card with the letter corresponding to the only correct answer.

You may refer to the following table during the exam.

Table of Laplace Transforms

$f(t)$	$F(s)$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$u_c(t)$	$\frac{e^{-cs}}{s}$
$u_c(t) \cdot f(t-c)$	$e^{-cs}F(s)$
$\delta(t-c)$	e^{-cs}

1. Find the general solution of the following differential equation, and determine how the solutions behave as $t \to \infty$.

$$ty' + 2y = 6, \qquad t > 0$$

$$y' + \frac{2}{t}y = \frac{6}{t} \qquad \mu(t) = e^{\int \frac{2}{t}dt} = e^{2\ln t} = e^{\ln t^2} = t^2$$

(A) $y \to -\infty$

(B) $y \to -6$

(C) $y \to -3$

(D) $y \to -2$

(E) $y \to 0$

(F) $y \to 2$

(G) $y \to 3$

(H) $y \to 6$

(I) $y \to \infty$

(J) The limit cannot be determined because it depends on the initial conditions.

$$t^2y' + 2ty = 6t$$

$$\int [t^2y' + 2ty]\,dt = \int 6t\,dt$$

$$t^2y = 3t^2 + C$$

$$y = 3 + \frac{C}{t^2}$$

as $t \to \infty$, $y \to 3$

2. Consider the initial value problem $(t^2 - 3t)y'' + ty' - (t+3)y = 0$, $y(1) = 2$, $y'(1) = 1$. The existence and uniqueness theorem for linear differential equations guarantees that there will be a unique solution. Determine the longest interval in which this solution is certain to exist.

(A) $-\infty < t < 0$

(B) $-\infty < t < 2$

(C) $-\infty < t < 3$

(D) $-\infty < t < \infty$

(E) $0 < t < 2$

(F) $0 < t < 3$

(G) $0 < t < \infty$

(H) $2 < t < 3$

(I) $2 < t < \infty$

(J) $3 < t < \infty$

$$y'' + \frac{t}{t^2-3t}y' - \frac{t+3}{t^2-3t}y = 0 \qquad t_0 = 1$$

discontinuous at $t = 0$ and $t = 3$

the longest interval containing $t_0 = 1$ throughout which the coefficient functions are continuous: $(0, 3)$

3. Find the general solution of the following differential equation.

$$y'' + 6y' + 9y = 0$$

$r^2 + 6r + 9 = 0$

$r = -3, -3$

(A) $y = c_1e^{3t} + c_2e^{3t}$
(B) $y = c_1e^{3t} + c_2t^3e^{3t}$
(C) $y = c_1e^{-3t} + c_2e^{-3t}$
(D) $y = c_1e^{-3t} + c_2te^{-3t}$
(E) $y = c_1e^{3t} + c_2e^{-3t}$
(F) $y = c_1t^3 + c_2t^3$
(G) $y = c_1t^3 + c_2t^3\ln t$
(H) $y = c_1t^{-3} + c_2t^{-3}$
(I) $y = c_1t^{-3} + c_2t^{-3}\ln t$
(J) $y = c_1t^3 + c_2t^{-3}$

4. Consider a spring system in which a 16 pound object stretches the spring six inches. (There is no damping.) What external force would cause resonance to occur?

$16 = m \cdot 32$ $\quad$ $16 = k \cdot \frac{1}{2}$

$m = \frac{1}{2}$ $\quad$ $k = 32$

$\frac{1}{2}u'' + 32u = 0$

$u'' + 64u = 0$

$r^2 + 64 = 0$

$r = \pm 8i$

$u = c_1 \cos 8t + c_2 \sin 8t$

natural frequency : 8

(A) $F(t) = t$
(B) $F(t) = e^t$
(C) $F(t) = te^t$
(D) $F(t) = e^{8t}$
(E) $F(t) = e^{-8t}$
(F) $F(t) = 2\cos(\sqrt{2}t)$
(G) $F(t) = \sin(\frac{4}{\sqrt{3}}t)$
(H) $F(t) = 5\cos(\frac{1}{4}t)$
(I) $F(t) = 6\sin(\frac{1}{\sqrt{6}}t)$
(J) $F(t) = 3\cos(8t) - \sin(8t)$