Differential Equations (Math 217) Exam 2 October 19, 2004

This exam contains ten multiple-choice problems worth two points each, five true-false problems worth one point each, and four problems to be hand-graded worth 25 points altogether, for an exam total of 50 points.

Part I. Multiple Choice. (2 points each)

For each of the following, choose the letter corresponding to the only correct answer.

1. What is the purpose of the method of reduction of order?

(A) Given a solution y_1 of a second-order linear *homogeneous* differential equation, find a second solution y_2 of the same differential equation such that the set $\{y_1, y_2\}$ is linearly *independent*.

(B) Given a solution y_1 of a second-order linear *homogeneous* differential equation, find a second solution y_2 of the same differential equation such that the set $\{y_1, y_2\}$ is linearly *dependent*.

(C) Given a solution y_1 of a second-order linear *nonhomogeneous* differential equation, find a second solution y_2 of the same differential equation such that the set $\{y_1, y_2\}$ is linearly *independent*.

(D) Given a solution y_1 of a second-order linear *nonhomogeneous* differential equation, find a second solution y_2 of the same differential equation such that the set $\{y_1, y_2\}$ is linearly *dependent*.

(E) Given a solution y_1 of a second-order linear *homogeneous* differential equation, find a solution y_2 of the corresponding *nonhomogeneous* equation.

(F) Given a solution y_1 of a second-order linear *nonhomogeneous* differential equation, find a solution y_2 of the corresponding *homogeneous* equation.

2. Consider the following fourth-order linear homogeneous differential equation with constant coefficients.

$$y^{(4)} - 6y''' + 9y'' + 4y' - 12y = 0$$

Find all the roots of the characteristic equation. How many *distinct real roots* are there? (In other words, how many roots are there if complex roots are not counted at all, and a real root of multiplicity k is counted only once, not k times.)

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

$$\begin{array}{r|rrrrr} 2 & 1 & -6 & 9 & 4 & -12 \\ & & 2 & -8 & 2 & 12 \\ \hline 2 & 1 & -4 & 1 & 6 & 0 \\ & & 2 & -4 & -6 & \\ \hline & 1 & -2 & -3 & 0 & \end{array}$$

$r = 2, 2, 3, -1$

$r^2 - 2r - 3 = 0$

$(r-3)(r+1) = 0$

3. Suppose y_1 and y_2 are solutions of a linear *nonhomogeneous* differential equation, and y_3 is a solution of the corresponding *homogeneous* equation. Then only two of the following are solutions of the *nonhomogeneous* equation. Which two are they?

(I) $y_1 + y_2$ (II) $y_1 + y_3$ (III) $y_2 - y_3$ (IV) $-y_2$ (V) $2y_1 + 5y_2$

(A) I and II
(B) I and III
(C) I and IV
(D) I and V
(E) II and III
(F) II and IV
(G) II and V
(H) III and IV
(I) III and V
(J) IV and V

Each solution of the nonhomogeneous equation must be the sum of a solution of the nonhomogeneous equation and a solution of the homogeneous equation. The Principle of Superposition applies only to homogeneous equations.

4. Suppose you are using the method of undetermined coefficients to solve the following linear nonhomogeneous differential equation.

$$y'' + 9y' = t^2 \cos 3t$$

How many terms are in the correct expression for the form of y_p?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5
(F) 6
(G) 7
(H) 8
(I) 9
(J) 10

$y_h = c_1 + c_2 e^{-9t}$

$y_p = At^2 \cos 3t + Bt^2 \sin 3t + Ct\cos 3t + Dt \sin 3t + E\cos 3t + F\sin 3t$

5. The method of undetermined coefficients can be used to solve which of the following differential equations?

(I) $t^2y'' - 4ty' + 4y = e^t$ – noncconstant coefficients
(II) $y'' - 4y' + 4y = e^t \sin 5t$
(III) $y'' - 4y' + 4y = \sec t$ – $f(t) = \sec t$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

(F) II and III only

(G) I, II, and III

(H) none of these

6. A spring oscillates according to the equation $\frac{d^2x}{dt^2} + 4x = 0$, where x is measured in feet and t is measured in seconds. Determine the period and the amplitude of these oscillations.

(A) $\frac{\pi}{2}$ sec., 1 ft.

(B) π sec., 1 ft.

(C) 2π sec., 1 ft.

(D) $\frac{\pi}{2}$ sec., 2 ft.

(E) π sec., 2 ft.

(F) 2π sec., 2 ft.

(G) $\frac{\pi}{2}$ sec., there is not enough data to determine the amplitude

(H) π sec., there is not enough data to determine the amplitude

(I) 2π sec., there is not enough data to determine the amplitude

(J) There are no oscillations.

$r^2 + 4 = 0 \quad r = \pm 2i$

$x = c_1 \cos 2t + c_2 \sin 2t$

period: $\frac{2\pi}{2} = \pi$

$x_0 = ? \quad v_0 = ?$