Differential Equations (Math 217) Exam 1 September 26, 2005

This exam contains twelve multiple-choice problems worth two points each, six true-false problems worth one point each, and three free-response problems worth 20 points altogether, for an exam total of 50 points.

Part I. Multiple Choice. (2 points each)

For each of the following, choose the letter corresponding to the only correct answer.

1. The general solution of the differential equation $x^2y'' = 2y$ is $y = Ax^{-1} + Bx^2$. (You do not need to verify this.) Find the solution of the following initial value problem.

$$x^2y'' = 2y \qquad y(1) = 3,\ y'(1) = 2$$

(A) $y = 3x^{-1}$

(B) $y = 3x^2$

(C) $y = 2x^{-1} + 3x^2$

(D) $y = 3x^{-1} + 2x^2$

(E) $y = \frac{1}{2}x^{-1} + \frac{5}{2}x^2$

(F) $y = \frac{5}{2}x^{-1} + \frac{1}{2}x^2$

(G) $y = \frac{1}{3}x^{-1} + \frac{8}{3}x^2$

(H) $y = \frac{8}{3}x^{-1} + \frac{1}{3}x^2$

(I) $y = \frac{4}{3}x^{-1} + \frac{5}{3}x^2$

(J) $y = \frac{5}{3}x^{-1} + \frac{4}{3}x^2$

$y = Ax^{-1} + Bx^2$

$y' = -Ax^{-2} + 2Bx$

$3 = A + B$

$2 = -A + 2B$

$5 = 3B$

$B = \frac{5}{3} \quad A = \frac{4}{3}$

$y = \frac{4}{3}x^{-1} + \frac{5}{3}x^2$

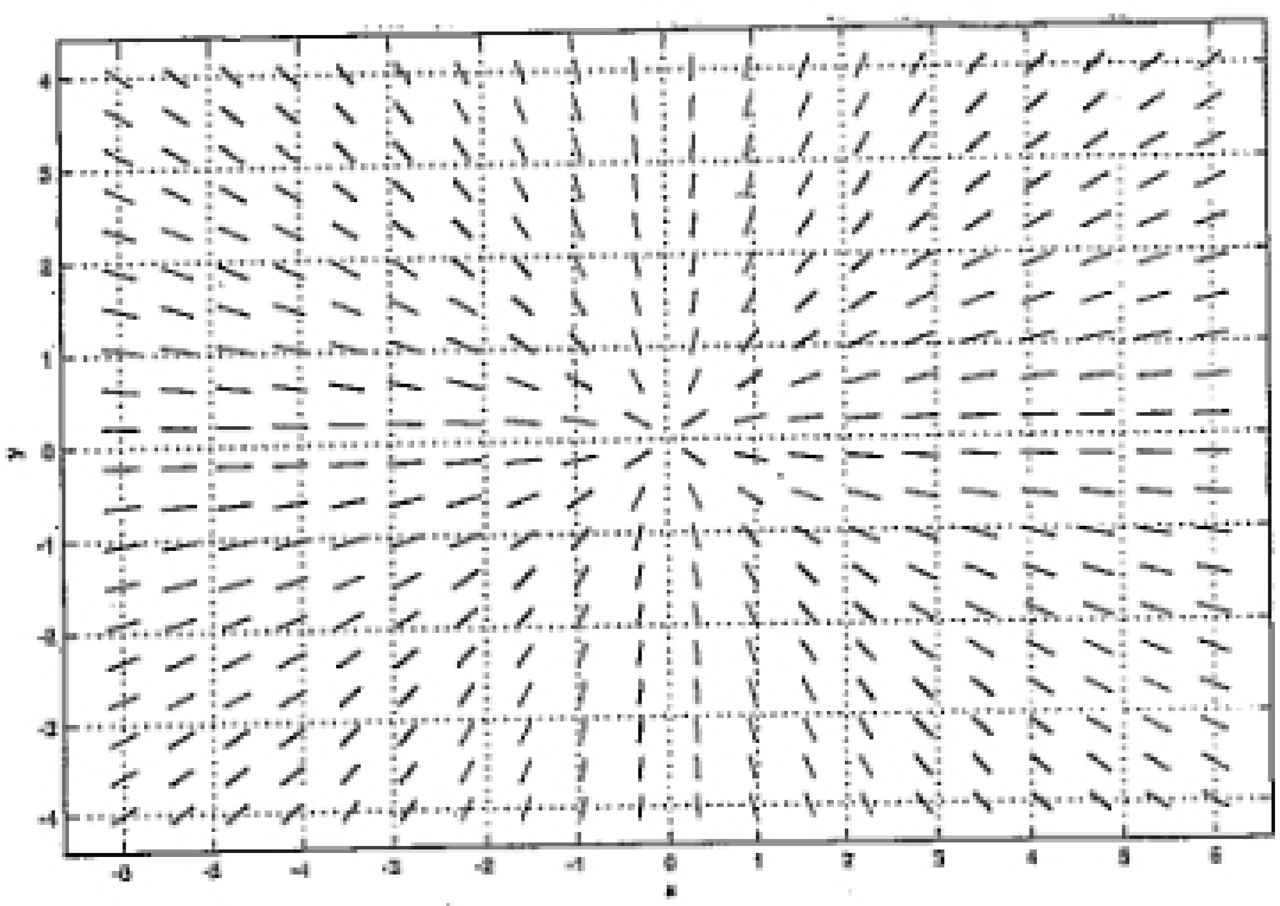
2. The direction field pictured corresponds to which of the following differential equations?

(A) $\frac{dy}{dx} = x + y$

(B) $\frac{dy}{dx} = xy$

(C) $\frac{dy}{dx} = \frac{x}{y}$

(D) $\frac{dy}{dx} = \frac{y}{x}$



For example, at $(3,-1)$, $m = \frac{dy}{dx} = \frac{y}{x} = \frac{-1}{3}$.

3. Solve the following initial value problem by separating variables and integrating.

$$\frac{dy}{dx} = x^3 e^{(\frac{1}{2}x^2)}, \quad y(0) = 0$$

What is the value of C? $\int dy = \int x^3 e^{\frac{1}{2}x^2} dx$ parts

$u = x^2 \quad dv = x e^{\frac{1}{2}x^2} dx$

$du = 2x\,dx \quad v = e^{\frac{1}{2}x^2}$

(A) $-\frac{1}{e}$

(B) $\frac{1}{e}$

(C) -1

(D) 1

(E) $-e^{\frac{1}{2}}$

(F) $e^{\frac{1}{2}}$

(G) -2

(H) 2

(I) $-e$

(J) e

$y = x^2 e^{\frac{1}{2}x^2} - \int 2x e^{\frac{1}{2}x^2} dx$

$y = x^2 e^{\frac{1}{2}x^2} - 2e^{\frac{1}{2}x^2} + C$

$y(0) = 0$

$0 = 0 - 2 + C \quad C = 2$

$y = x^2 e^{\frac{1}{2}x^2} - 2e^{\frac{1}{2}x^2} + 2$

4. Find an integrating factor which could be used to solve the following linear differential equation.

$$(x^2+1)\frac{dy}{dx} - 2xy = 4$$

(A) $\frac{1}{x^2+1}$

(B) x^2+1

(C) $(x^2+1)^2$

(D) e^{-x^2}

(E) e^{x^2}

(F) e^{x^2+1}

(G) $\ln(x^2)$

(H) $\ln(x^2+1)$

(I) $\frac{1}{\ln(x^2+1)}$

$\frac{dy}{dx} - \frac{2x}{x^2+1} y = \frac{4}{x^2+1}$

$\mu(x) = e^{\int \frac{-2x}{x^2+1} dx} = e^{-\ln(x^2+1)}$

$= e^{\ln(x^2+1)^{-1}} = (x^2+1)^{-1}$

5. The following differential equation is exact. (You do not need to verify this.) Find its general solution.

$$\underbrace{e^x\sin(e^y)}_{M}dx + \underbrace{e^x e^y\cos(e^y)}_{N}dy = 0$$

(A) $y = e^x\cos(e^y)$

(B) $y = e^x\sin(e^y)$

(C) $y = e^x\sin(e^y)\cos(e^y)$

(D) $F = e^x\cos(e^y) + C$

(E) $F = e^x\sin(e^y) + C$

(F) $F = e^x\sin(e^y)\cos(e^y) + C$

(G) $e^x\cos(e^y) = C$

(H) $e^x\sin(e^y) = C$

(I) $e^x\sin(e^y)\cos(e^y) = C$

$$F = \int e^x\sin(e^y)dx = \sin(e^y)\int e^x dx$$
$$= \sin(e^y)\cdot e^x + g(y)$$
$$\frac{\partial F}{\partial y} = \cos(e^y)\cdot e^y\cdot e^x + g'(y)$$
$$= e^x e^y\cos(e^y)$$
$$g'(y) = 0 \quad g(y) = 0$$
$$F = e^x\sin(e^y)$$

solution: $e^x\sin(e^y) = C$

6. The following differential equation can be put into the proper form and solved as which of the following types?

$$\frac{dy}{dx} = \frac{x^2-y^2}{2xy}$$

(I) separable (II) linear in x (III) linear in y (IV) exact

(A) none of these

(B) I only

(C) II only

(D) III only

(E) IV only

(F) I and II only

(G) I and III only

(H) I and IV only

(I) II and IV only

(J) III and IV only

$$2xy\,dy = (x^2-y^2)dx$$
$$(y^2-x^2)dx + 2xy\,dy = 0$$
$$\frac{\partial M}{\partial y} = 2y \quad \frac{\partial N}{\partial x} = 2y \quad \text{exact}$$

not separable because of the factor x^2-y^2

not linear in x because of x^2

not linear in y because of y^2