Midterm 1, Math 217
Sep. 25th, 2006

Name:______________ Student ID: ______________ Score: ______________

Part I: Multiple Choices(5pts each)

(1)Classify the differential equation $e^t y'' - 5t^2 y = sin(y)$.

(A)ordinary, linear, order 1

(B)ordinary, linear order 2

(C)ordinary, nonlinear, order 1

(D)ordinary, nonlinear, order 2

(E)partial, linear, order 1

(F)partial, linear, order 2

(G)partial, nonlinear, order 1

(H)partial, nonlinear, order 2

(D) Because sin(y) is not linear.

2.Suppose that $M(x, y)$ is the partial derivative of $F(x, y)$ with respect to x and that $N(x, y)$ is the partial derivative of $F(x, y)$ with respect to y. Assume that all of these functions have continuous derivatives of all orders. Which of the following statements best describes the equation $\frac{d}{dx}y(x) = -\frac{M(x,y)}{N(x,y)}$? (In the answers, C is a constant.)

(A)The equation is separable.

(B)The equation is homogeneous.

(C)The equation is linear.

(D)The solution is give implicitly by $\int N(x, y)dy = -\int M(x, y)dx + C$.

(E)The solution is given explicitly as $y(x) = F(x, C)$.

(F)The solution is given implicitly by $x = F(C, y(x))$.

$\boxed{\text{(F)}}$The solution is given implicitly by $C = F(x, y(x))$.

(G)The solution is given implicitly by $y(x) = F(x, y(x))$.

(H)The solution has no solution.

$\frac{dy}{dx} = -\frac{F_x}{F_y}$

$F_y dy = -F_x dx$

$F_y dy + F_x dx = 0$

$\frac{dF}{dx} = 0.$

$F(x, y(x)) = C.$

6.Find the solution of the given initial value problem.

$$y' + \frac{2}{t}y = \frac{\cos(t)}{t^2}, \quad y(\pi) = 0,\ t > 0$$

(C)

(A)$\frac{\sin(t)}{t}$,

(B)$\frac{\sin(t^2)}{t}$,

(C)$\frac{\sin(t)}{t^2}$,

(D)$\frac{\sin t^2}{t^2}$,

(E)$\frac{\cos(t)}{t}$,

(F)$\frac{\cos(t^2)}{t}$,

(G)$\frac{\cos(t)}{t^2}$,

(H)$\frac{\cos(t^2)}{t^2}$.

$t^2y' + 2ty = \cos(t)$

$(t^2y)' = \cos(t)$

$t^2y = \sin(t) + C$

$y = \frac{\sin(t)}{t^2} + \frac{C}{t^2}$

$y(\pi) = 0 + \frac{C}{\pi^2} = 0 \quad C = 0$

$y = \frac{\sin(t)}{t^2}$

7.Find the limit $\lim_{t\to\infty} y(t)$? where $y(t)$ is the solution to the initial value problem

$$2y' + 5y = 2, \quad y(0) = 1.$$

(A)0

(B)1

(C)2

(D)3

(E)does not exist.

$y' + \frac{5}{2}y = 1$

$\mu(t) = e^{\frac{5}{2}t}$

$y = \frac{1}{e^{\frac{5}{2}t}} \int e^{\frac{5}{2}t}\,dt = \frac{1}{e^{\frac{5}{2}t}}\left(\frac{2}{5}e^{\frac{5}{2}t} + C\right)$

$= \frac{2}{5} + \frac{C}{e^{\frac{5}{2}t}}$

$\lim_{t\to\infty} y(t) = \frac{2}{5}$

$y(0) = 1 = \frac{2}{5} + C \quad C = \frac{3}{5}$

8.What is the general solution to $\frac{dy}{dx} = 2y(y+3)$?

(E)

(A)$y = Ce^{x}$,

(B)$y = \frac{2Ce^{3t}}{1-Ce^{3t}}$

(C)$y = \frac{2Ce^{3t}}{1+Ce^{3t}}$

(D)$y = \frac{2Ce^{3t}}{1\pm Ce^{3t}}$

(E)$y = \frac{3Ce^{6t}}{1-C^{6t}}$

(F)$y = \frac{3Ce^{6t}}{1+C^{6t}}$

(G)$y = \frac{3Ce^{6t}}{1\pm C^{6t}}$

(H)none of the above

$\frac{dy}{y(y+3)} = 2dx$

$\frac{1}{3}\left(\frac{1}{y} - \frac{1}{y+3}\right)dy = 2dx$

$\frac{1}{3}(\ln y - \ln(y+3)) = 2x + C$

$\ln\left|\frac{y}{y+3}\right| = 6x + C$

$\frac{y}{y+3} = Ce^{6x}$

$y = yCe^{6x} + 3Ce^{6x}$

$y(1 - Ce^{6x}) = 3Ce^{6x}$

$y = \frac{3Ce^{6x}}{1 - Ce^{6x}}$

12.How many solutions does the equation $y' = \frac{\cos(t)y^2}{t+y^3}$ have satisfying the initial value $y(0) = 1$?

(B)

(A)0

(B)1

(C)2

(D)3

(E)infinite

13.How many solutions does the initial value problem have?

$$y' = y^{1/3}, \quad y(0) = 0.$$

(E)

(A)0

(B)1

(C)2

(D)3

(E)infinite

14.Suppose a certain population satisfies the initial value problem

$$\frac{dP}{dt} = 50P(6000 - P), P(0) = 2000.$$

Choose the statement that correctly describe the concavity of the solution curve.

(A)The graph is concave up everywhere.

(D)

(B)The graph is concave down everywhere.

(C)There is an inflection point where $P = 2000$.

(D)There is an inflection point where $P = 3000$.

(E)There is an inflection point where $P = 3500$.

(F)None of the above.

15.Which one of the following equations is exact?

(C)

(A)$(2x + 4y) + (2x - 2y)y' = 0$.

(B)$(x\ln(y) + xy)dx + (y\ln(x) + xy)dy = 0$.

(C)$\frac{dy}{dx} = -\frac{ax+by}{bx+cy}$.

(D)$\frac{dy}{dx} = -\frac{ax-by}{bx-cy}$.

$(bx+cy)dy = -(ax+by)dx$

$(bx+cy)dy + (ax+by)dx = 0$

$\frac{\partial(bx+cy)}{\partial x} = b$

$\frac{\partial(ax+by)}{\partial y} = b$