EE 428 **EXAM I** **17 October 2006**

Last Name (Print): Solutions

First Name (Print): ______________________

ID number (Last 4 digits): ____________

Section: ___

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Problem	Weight	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

INSTRUCTIONS

1. You have 2 hours to complete this exam.
2. This is a closed book exam. You may use one 8.5" × 11" note sheet.
3. Calculators are allowed.
4. Solve each part of the problem in the space following the question. If you need more space, continue your solution on the reverse side labeling the page with the question number; for example, **Problem 1.2 Continued**. **NO** credit will be given to solutions that do not meet this requirement.
5. **DO NOT REMOVE ANY PAGES FROM THIS EXAM.** Loose papers will not be accepted and a grade of **ZERO** will be assigned.
6. The quality of your analysis and evaluation is as important as your answers. Your reasoning must be precise and clear; your complete English sentences should convey what you are doing. **To receive credit, you must show your work.**

Problem 1: (25 Points)

Control engineers are often confronted with a physical system for which the input-output model is unknown. In situations where there is insufficient information to derive a model from first principles (KVL,KCL, Newton's second law, etc.), it may be possible to estimate a model from input-output data. The use of experimental observations to determine a system model is called **system identification.** Many techniques have been developed for identifying a system through analysis of input-output data. This problem considers two approaches in which the system is excited by a known input function and the response is used to estimate the coefficients of a differential equation that will serve as an input-output model of the system.

1. (10 points) The zero-state unit-step response of a certain SISO LTI system is

$$y(t) = \left\{1 - e^{-4t}(1 + 4t)\right\} 1(t).$$

Based on this input-output data, find an ODE representation of the system and place your answer in the standard form

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \cdots + a_o y = b_m \frac{d^m u}{dt^m} + b_{m-1}\frac{d^{m-1}u}{dt^{m-1}} + \cdots + b_o u.$$

We can determine the transfer function representation of the system as:

$H(s) = \mathcal{L}\{y(t)\} / \mathcal{L}\{u(t)\}$ where

$$\begin{cases} y(t) = 1(t) - e^{-4t}1(t) - 4te^{-4t}1(t) \\ Y(s) = \frac{1}{s} - \frac{1}{s+4} - \frac{4}{(s+4)^2} \end{cases}$$

$$\begin{cases} u(t) = 1(t) \\ U(s) = \frac{1}{s} \end{cases}$$

And so

$$H(s) = \frac{Y(s)}{U(s)} = \frac{\frac{1}{s} - \frac{1}{s+4} - \frac{4}{(s+4)^2}}{\frac{1}{s}} = 1 - \frac{s}{s+4} - \frac{4s}{(s+4)^2}$$

$$= \frac{(s+4)^2 - s(s+4) - 4s}{(s+4)^2} = \frac{\cancel{s^2} + \cancel{8s} + 16 - \cancel{s^2} - \cancel{4s} - \cancel{4s}}{s^2 + 8s + 16}$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{16}{s^2 + 8s + 16}}$$

As expected, the DC gain $H(0) = 1$ as $\lim_{t\to\infty} y(t) = 1$.

Cross-multiplying yields

$$s^2 Y(s) + 8s Y(s) + 16 Y(s) = 16 U(s)$$

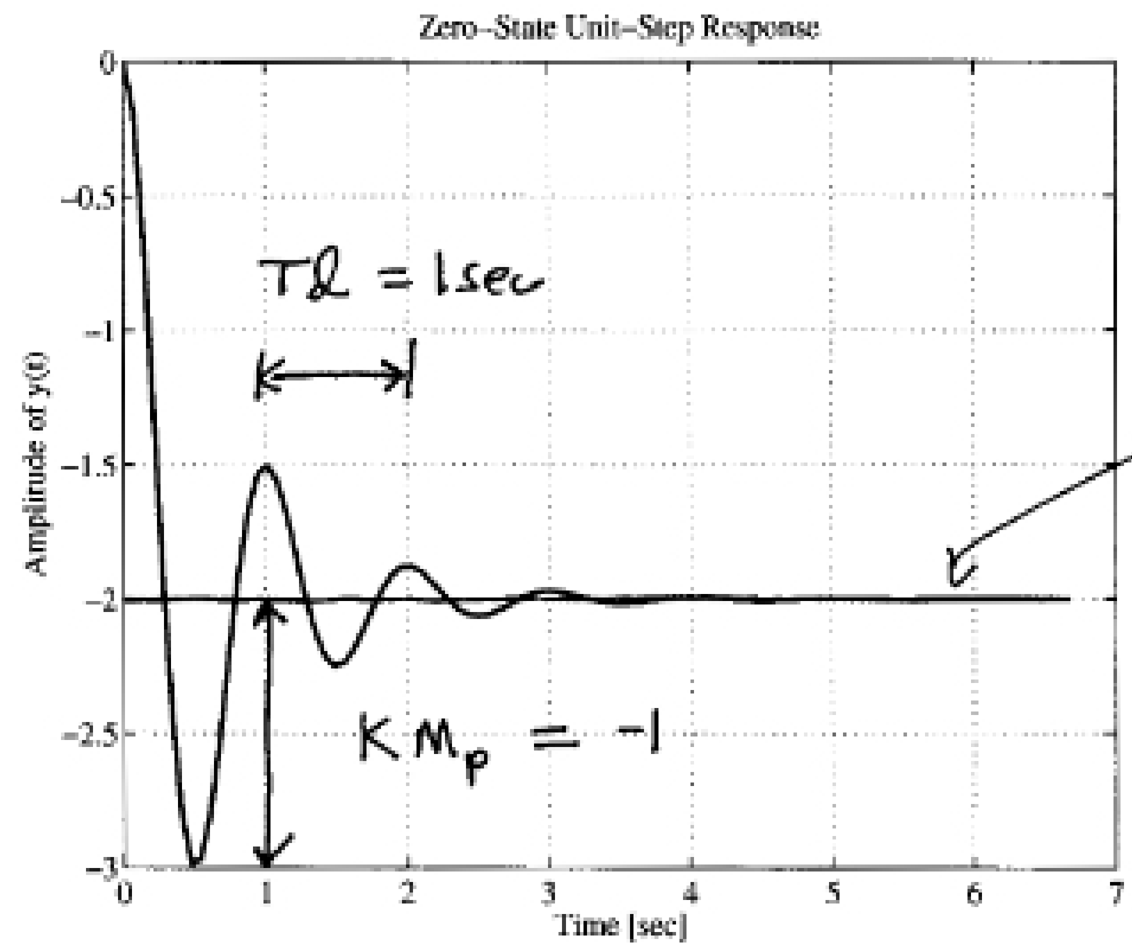
and so

$$\boxed{\ddot{y} + 8\dot{y} + 16y = 16u}$$

2. (15 points) Figure 1 shows the zero-state unit-step-response of another SISO LTI system with input $u(t)$ and output $y(t)$. Based on the input-output data, can the ODE model

$$\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0y(t) = b_o u(t) \tag{1}$$

represent the dynamics of the system? If so, specify the numeric values of the coefficients a_1, a_o, and b_o. If not, explain why.



steady-state value of -2 indicates DC gain of -2

Figure 1: Zero-state unit-step response of a SISO LTI system.

From Figure 1:

- DC gain $= K = -2$
- $K m_p = -1 \Rightarrow m_p = 0.5$
- $\omega_d = \frac{2\pi}{T_d} = 2\pi$

Using

$$m_p = e^{-\pi\varphi/\sqrt{1-\varphi^2}},$$

$$\varphi = \frac{|\ln(m_p)|}{\sqrt{\ln^2(m_p) + \pi^2}} = 0.2155.$$

It follows that

$$\omega_n = \frac{\omega_d}{\sqrt{1-\varphi^2}} = \frac{2\pi}{\sqrt{1-(0.2155)^2}} = 6.4344$$

And so

$$\boxed{\begin{aligned} a_1 &= 2\varphi\omega_n = 2.773 \\ a_0 &= \omega_n^2 = 41.4 \\ b_0 &= K\omega_n^2 = -82.8 \end{aligned}}$$