

Last Name (Print): Solutions

First Name (Print): _____

ID number (Last 4 digits): _____

Section: _____

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Problem	Weight	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

INSTRUCTIONS

1. You have 2 hours to complete this exam.
2. This is a closed book exam. You may use two 8.5" × 11" note sheet.
3. Calculators are allowed.
4. Solve each part of the problem in the space following the question. If you need more space, continue your solution on the reverse side labeling the page with the question number; for example, **Problem 1.2 Continued**. **NO** credit will be given to solutions that do not meet this requirement.
5. **DO NOT REMOVE ANY PAGES FROM THIS EXAM.** Loose papers will not be accepted and a grade of **ZERO** will be assigned.
6. The quality of your analysis and evaluation is as important as your answers. Your reasoning must be precise and clear; your complete English sentences should convey what you are doing. **To receive credit, you must show your work.**

Problem 1: (25 Points)

Figure 1 shows a closed-loop system with nonunity feedback, where

$$G(s) = \frac{10(s+10)}{s(s+2)}$$

and

$$H(s) = s+4.$$

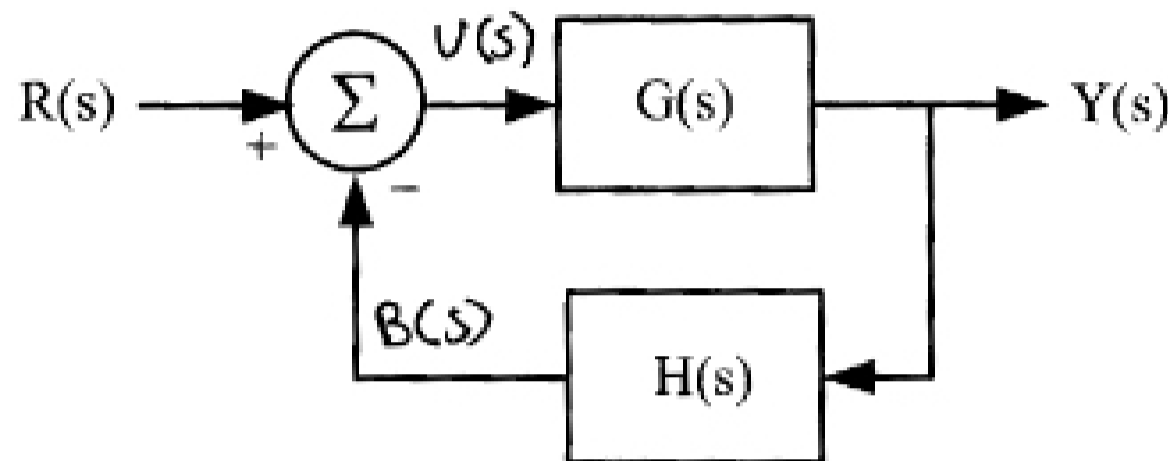
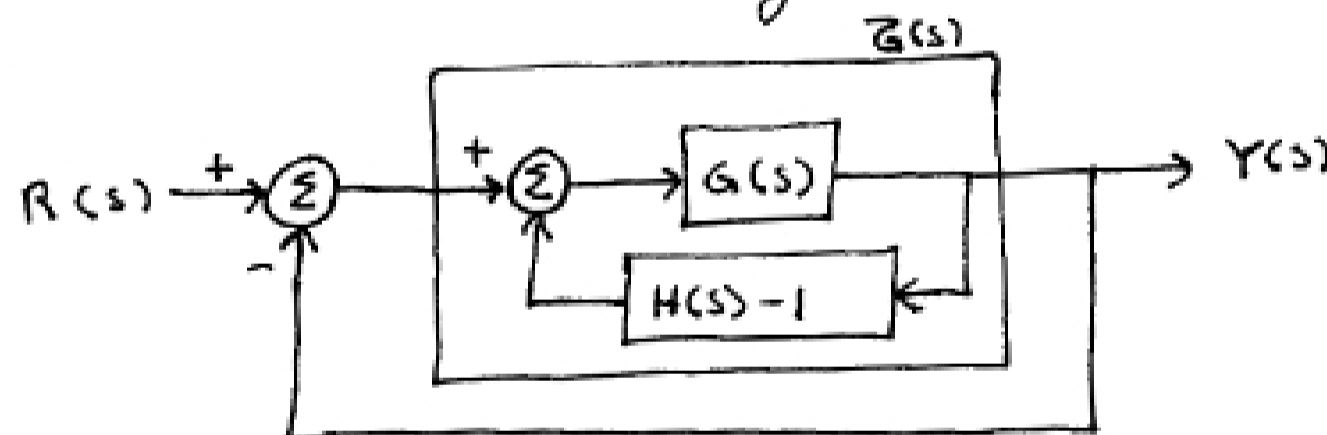


Figure 1: Closed-loop system with nonunity feedback .

Determine

1. (5 points) the system type,
2. (5 points) the value of the finite, nonzero error constant,
3. (5 points) the input waveform that yields a constant, nonzero steady-state error,
4. (5 points) the steady-state error for the waveform in part 3, and
5. (5 points) the steady-value of the actuating signal.

1. The system is determined by both $G(s)$ and $H(s)$. Redraw the system



where

$$\bar{G}(s) = \frac{G}{1+(H-1)G} = \frac{\frac{10(s+10)}{s(s+2)}}{1+(s+4-1)\frac{10(s+10)}{s(s+2)}} = \frac{10(s+10)}{s(s+2)+10(s+3)(s+10)}$$

$$\bar{G}(s) = \frac{10(s+10)}{11s^2+132s+300}$$

Is the closed-loop system BIBO stable? Consider the closed-loop system

$$\frac{Y}{R} = \frac{\bar{G}}{1+\bar{G}} = \frac{10(s+10)}{11s^2 + 132s + 300 + 10s + 100}$$

Because Y/R is second-order and all the coefficients are positive, the system is BIBO stable

1. Because $\bar{G}(s)$ has no poles at the origin, the system type is 0.

2. For a type 0 system, only the position error constant is non zero and finite:

$$K_p = \lim_{s \rightarrow 0} \bar{G}(s) = \lim_{s \rightarrow 0} \frac{10(s+10)}{11s^2 + 132s + 300} = \frac{1}{3} \quad \boxed{K_p = \frac{1}{3}}$$

3. For a type 0 system, only a constant input (step waveform) yields a constant error.

$$4. e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\frac{1}{3}} = \frac{1}{\frac{4}{3}} \quad \boxed{e_{ss} = \frac{3}{4}}$$

5. The actuator signal, $U(s)$, appears at the input of $G(s)$ as shown in Figure 1. To achieve a steady-state error, $y(t)$ must approach a constant. Because $G(s)$ has a pole at the origin, it behaves as an integrator. As a result, $y(t)$ can only approach a constant if the input $u(t)$ approaches 0. Conclude $U_{ss} = 0$.

Alternatively, $y_{ss} = r_{ss} - e_{ss} = 1 - \frac{3}{4} = \frac{1}{4}$. Because $H(0) = 4$, the steady-state value of $b(t)$ (indicated in Figure 1) is $b_{ss} = H(0)y_{ss} = 4 \cdot \frac{1}{4} = 1$. It follows that $U_{ss} = r_{ss} - b_{ss} = 1 - 1 = 0$.