1. Which of the following equations has a regular singular point at $x = 0$?

I) $xy'' + (1-x)y' + xy = 0$ II) $x^2(1-x)^2 y'' + 2xy' + 4y = 0$

III) $x^2(1-x)y'' + (x-2)y' - 3xy = 0$

A) I

B) II

C) III

D) I, II

E) I, III

F) II, III

G) All of them

H) None of them

I) $P = x$, $Q = 1-x$, $R = x$

$\lim_{x \to 0} x \frac{Q}{P} = 1$, $\lim x^2 \frac{R}{P} = 0$ REG.

II) $P = x^2(1-x)^2$, $Q = 2x$, $R = 4$

$\lim x \frac{Q}{P} = 2$, $\lim x^2 \frac{R}{P} = 4$ REG.

III) $P = x^2(1-x)$, $Q = x-2$, $R = -3x$

$\lim_{x \to 0} x \frac{Q}{P} = \lim x \frac{x-2}{x^2(1-x)}$

$= \lim \frac{x-2}{x(1-x)} = -\infty$ NOT REG.

$\lim_{x \to 0} x^2 \frac{R}{P} = \lim x^2 \frac{-3x}{x^2(1-x)}$

$= \lim \frac{-3x}{(1-x)} = 0$

2. Find the general solution of the equation $t^2 y'' + 4ty' + 2y = 0, \quad t > 0$.

A) $At + Bt\ln(t)$

B) $At^{-1} + Bt\ln(t)$

C) $At^{-1} + Bt^{-2}$

D) $At + Bt^{-1}$

E) $At\cos(2\ln t) + Bt\sin(2\ln t)$

F) $At\cos(\ln t) + Bt\sin(\ln t)$

G) $At^{-1}\cos(2\ln t) + Bt^{-1}\sin(2\ln t)$

H) $At^{-1}\cos(\ln t) + Bt^{-1}\sin(\ln t)$

I) $At^{-1} + Bt^{2}$

J) None of the above

EULER EQ'N,

$\alpha = 4, \ \beta = 2$

FIND) $r^2 + 3r + 2 = 0$

$(r+1)(r+2) = 0$

$r = -1, \quad r = -2$

$y(t) = c_1 t^{-1} + c_2 t^{-2}$

3. Find the general solution of the equation $t^2y'' + 3ty' + 5y = 0, \quad t > 0$.

A) $At + Bt\ln(t)$

B) $At^{-1} + Bt\ln(t)$

C) $At^{-1} + Bt^{-2}$

D) $At + Bt^{-1}$

E) $At\cos(2\ln t) + Bt\sin(2\ln t)$

F) $At\cos(\ln t) + Bt\sin(\ln t)$

G) $At^{-1}\cos(2\ln t) + Bt^{-1}\sin(2\ln t)$

H) $At^{-1}\cos(\ln t) + Bt^{-1}\sin(\ln t)$

I) $At^{-1} + Bt^2$

J) None of the above

EULER EQN

$\alpha = 3, \quad \beta = 5$

*(IND) $r^2 + 2r + 5 = 0$

$r = -1 \pm 2i$

so $\lambda = -1, \ \mu = 2.$

$y(t) = c_1 t^{-1}\cos(2\ln t) + c_2 t^{-1}\sin(2\ln t)$