1. Which of the following matrices have real eigenvalues?

I) $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ II) $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ III) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

A) I

B) II

C) III

D) I, II

E) I, III

F) II, III

G) I, II, III

H) None of them

IN EACH CASE, COMPUTE

$\det(A - r I)$, SET $= 0$.

I) $0 = \det\begin{pmatrix} 1-r & 0 \\ 0 & 2-r \end{pmatrix}$

$= (1-r)(2-r)$

SO $r = 1, 2$ BOTH REAL.

II) $0 = \det\begin{pmatrix} -r & 1 \\ 1 & 1-r \end{pmatrix}$

$= -r(1-r) - 1 = r^2 - r - 1$

SO $r = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$ REAL

III) $0 = \det\begin{pmatrix} -r & -1 \\ 1 & -r \end{pmatrix}$

$= r^2 + 1$ SO $r = \pm i$ COMPLEX

2. Let $w(t) = W\left(\begin{pmatrix} t \\ 1 \end{pmatrix}, \begin{pmatrix} t^2 \\ 2t \end{pmatrix}\right)$, the Wronskian of the vector functions $\begin{pmatrix} t \\ 1 \end{pmatrix}$ and $\begin{pmatrix} t^2 \\ 2t \end{pmatrix}$. Find w(2).

A) 0

B) 1/3

C) 1/2

D) 1

E) 3/2

F) 2

G) 5/2

H) 3

I) 4

J) None of the above

$$w(t) = \det\begin{pmatrix} t & t^2 \\ 1 & 2t \end{pmatrix}$$

$$= 2t^2 - t^2 = t^2$$

$$\text{so } \quad w(2) = 2^2 = 4$$

3. Which of the following vector functions $\vec{x}(t)$ are solutions of $\vec{x}' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}\vec{x}$?

I) $\begin{pmatrix} e^{-t} \\ e^{-t} \end{pmatrix}$ II) $\begin{pmatrix} e^{-3t} \\ -e^{-3t} \end{pmatrix}$ III) $\begin{pmatrix} e^{t} \\ 3e^{t} \end{pmatrix}$

A) I

B) II

C) III

D) I, II

E) I, III

F) II, III

G) I, II, III

H) None of them

FIND G.S. OF $*)$.

E-VALUES OF $\begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$:

$$0 = \det\begin{pmatrix} -2-r & 1 \\ 1 & -2-r \end{pmatrix} = \dots =$$

$$r^2+4r+3 = (r+1)(r+3)$$

SO $r = -1, -3$

EVECS

$\vec{\xi} \leftrightarrow r=-1$ SOLVE $(A-(-1)I)\vec{\xi} = 0$

OR $\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \xi_1 = \xi_2$

SO TAKE $\vec{\xi} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\vec{\xi} \leftrightarrow r=-3$ SOLVE $(A-(-3)I)\vec{\xi} = 0$

OR $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\vec{\xi} = \vec{0} \Rightarrow \xi_2 = -\xi_1$

SO TAKE $\vec{\xi} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

G.S. OF $*)$ IS $c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{-t} + c_2\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-3t}$

TO ANSWER QUESTION

$\begin{pmatrix} e^{-t} \\ e^{-t} \end{pmatrix}$, $\begin{pmatrix} e^{-3t} \\ -e^{-3t} \end{pmatrix}$ ARE OF THIS FORM,

$\begin{pmatrix} e^{t} \\ 3e^{t} \end{pmatrix}$ IS NOT.