33MATH 132 EXAMINATION I FALL 2008

SOLUTIONS

PART I : (80 points)

1. Given that $\frac{dy}{dx} = \frac{1}{2}e^{2x} + \frac{1}{2}$ and that $y(0) = 1$, find y(1).

A) $\frac{2}{3}e^{\frac{3}{2}} + \frac{1}{3}$ B) $\frac{2e^2+3}{4}$ C) $\frac{1}{2}e^2 + \frac{2}{3}$ D) $\frac{e^2-8}{3}$ E) $\frac{2}{5}e^3 + \frac{1}{3}$

F) $\frac{1}{2}e^3 + 2$ G) $\frac{e^2+5}{4}$ H) $\frac{1}{5}e^2 + \frac{3}{2}$ I) $\frac{e^3+6}{4}$ J) $\frac{1}{2}e^3 + \frac{1}{2}$

Solutions: $y=\frac{1}{4}e^{2x} + \frac{1}{2}x + C$. $1 = y(0) = \frac{1}{4} + C$, so $C = \frac{3}{4}$ and $y=\frac{1}{4}e^{2x} + \frac{1}{2}x + \frac{3}{4}$. Then $y(1) = \frac{1}{4}e^2 + \frac{5}{4} = \frac{e^2+5}{4}$. (G)

2. Approximate the **area** under the curve $y = \frac{1}{x}$, $1 \leq x \leq 3$, using the **left** point Riemann sum with $n = 4$.

A) 1.0986 B) 1.1136 C) 1.1478 D) 1.1665 E) 1.1879

F) 1.2135 G) 1.2472 H) 1.2651 I) 1.2833 J) 1.3091

Solution: $\Delta x = \frac{1}{2}$ $L_4 = (1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{5})(\frac{1}{2}) = 1.2833333...$ (I)

3) For $y = \int_1^{x^3} \sqrt[3]{t}\, dt$, find the value of $\frac{dy}{dx}$ at $x = 2$.

A) 12 B) 24 C) 32 D) 44 E) 15 F) 36 G) 47 H) 28 I) 39 J) 33

Solution: $\frac{dy}{dx} = \sqrt[3]{x^3}\,(3x^2)$. At $x = 2$ $\frac{dy}{dx} = 2 \cdot 3 \cdot 4 = 24$ (B)

4. Evaluate $\int_0^{ln(2)} e^{3x}\, dx$.

A) $\frac{2}{3}$ B) $\frac{3}{4}$ C) $\frac{4}{3}$ D) $\frac{3}{4}$ E) $\frac{7}{3}$ F) $\frac{4}{5}$ G) $\frac{9}{5}$ H) $\frac{7}{4}$ I) $\frac{9}{2}$ J) $\frac{9}{4}$

Solution: $= \frac{1}{3}e^{3x}\Big|_0^{ln(2)} = \frac{1}{3}((e^{ln(2)})^3 - 1) = \frac{1}{3}(8-1) = \frac{7}{3}$. (E)

5) Evaluate $\int_0^a x\sqrt{a^2 - x^2}\,dx$ using the substitution $u = a^2 - x^2$ (a is some constant).

A) 0 B) a C) a^2 $D)\, a^3$ $E)\, \frac{a}{2}$ $F)\, \frac{a^2}{3}$ $G)\, \frac{a^3}{3}$ $H)\, \frac{a^4}{5}$ $I)\, a^2 - a$

Solution: (du=-2x dx) $= -\frac{1}{2}\int_{a^2}^{0} u^{\frac{1}{2}} du = \frac{1}{3}\, u^{\frac{3}{2}}\Big|_0^{a^2} = \frac{1}{3}\, a^3$. (G)

6. Evaluate $\int_0^{\frac{\pi}{4}} tan^2(x)\, sec^4(x)\, dx$ using the substitution $u = tan(x)$. (Hint:Also needs a trig. identity)

A) $\frac{1}{15}$ B) $\frac{2}{15}$ $C)\, \frac{1}{5}$ $D)\, \frac{4}{15}$ $E)\, \frac{1}{3}$ $F)\, \frac{5}{6}$ $G)\, \frac{7}{15}$ $H)\, \frac{8}{15}$ $I)\, \frac{3}{5}$ $J)\, \frac{2}{3}$

Solution: (u= tan(x) du= $sec^2(x)$, *also* $sec^2(x) = 1 + tan^2(x)$)
$= \int_0^1 u^2(1 + u^2)\, du = \int_0^1 u^2 + u^4\, du = \frac{u^3}{3} + \frac{u^5}{5}\Big|_0^1 = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$. (H)

7) Evaluate the indefinite integral $\int \frac{1}{x^2} sin(\frac{1}{x})\, cos(\frac{1}{x})\, dx$.

A) $\frac{1}{2} cos(\frac{1}{x}) + C$ $B)\, \frac{1}{2} sin(\frac{1}{x}) + C$ $C)\, \frac{1}{2} cos^2(\frac{1}{x}) + C$ $D)\, sin(\frac{1}{x})\, cos(\frac{1}{x}) + C$

E) $2\, cos^2(\frac{1}{x}) + C$ $F)\, sin^2(\frac{1}{x})\, cos^2(\frac{1}{x}) + C$ G) $2\sin(\frac{1}{x}) + C$

H) $4\, cos^2(\frac{1}{x}) + C$ I) $2\, sin(\frac{1}{x})\, cos(\frac{1}{x}) + C$

Solution: (u= $cos(\frac{1}{x})$ $du = -\,sin(\frac{1}{x}) \cdot (-\frac{1}{x^2})\, dx = (\frac{1}{x^2}) sin(\frac{1}{x})\, dx$)
$\int \frac{1}{x^2} sin(\frac{1}{x})\, cos(\frac{1}{x})\, dx = \int u\, du = \frac{1}{2}u^2 + C = \frac{1}{2} cos^2(\frac{1}{x}) + C$. (C)

8. Evaluate $\int_4^8 \frac{\log_2(x)}{x}\, dx$.

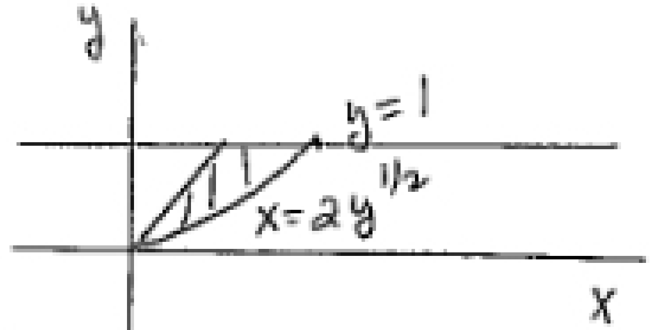
A) 1.7329 $B)\, 1.8112$ $C)\, 2.1357$ $D)\, 2.5481$ $E)\, 2.9765$ $F)\, 3.3652$ $G)\, 3.5672$

H) 3.7563 $I)\, 3.9865$

Solution: (u= $\log_2(x)$ du=$\frac{1}{ln(2)} \cdot \frac{1}{x}\, dx$)
$\int_4^8 \frac{\log_2(x)}{x}\, dx = ln(2)\int_2^3 u\, du = \frac{ln(2)}{2}\, u^2\Big|_2^3 = \frac{5\,ln(2)}{2} = 1.7328679$. (A)

9. Find the area bounded on the right by the curve $y = \frac{1}{4}x^2$, on the left by the line $y = x$ and above by the line $y = 1$.

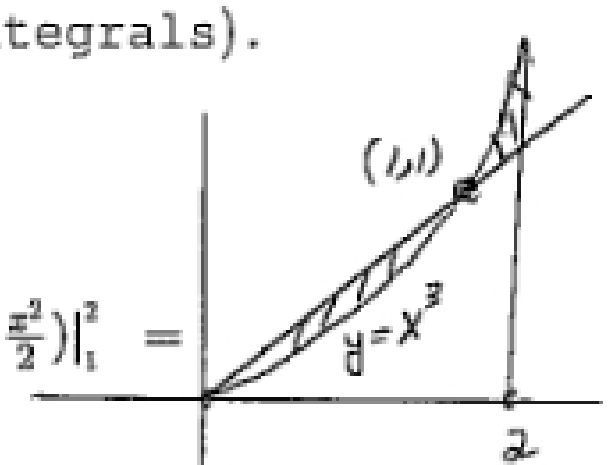
A) $\frac{1}{15}$ B) $\frac{2}{15}$ C) $\frac{1}{5}$ D) $\frac{4}{15}$ E) $\frac{1}{3}$ F) $\frac{5}{6}$ G) $\frac{7}{15}$ H) $\frac{8}{15}$ I) $\frac{3}{5}$ J) $\frac{2}{3}$



Solutions: $A = \int_0^1 2y^{\frac{1}{2}} - y\, dy = \frac{4}{3}y^{\frac{3}{2}} - \frac{1}{2}y^2 \big|_0^1 = \frac{4}{3} - \frac{1}{2} = \frac{5}{6}$ (F)

10. Find the area of the region enclosed by $y = x^3$ and $y = x$, $0 \leq x \leq 2$ (Hint: you'll need 2 integrals).

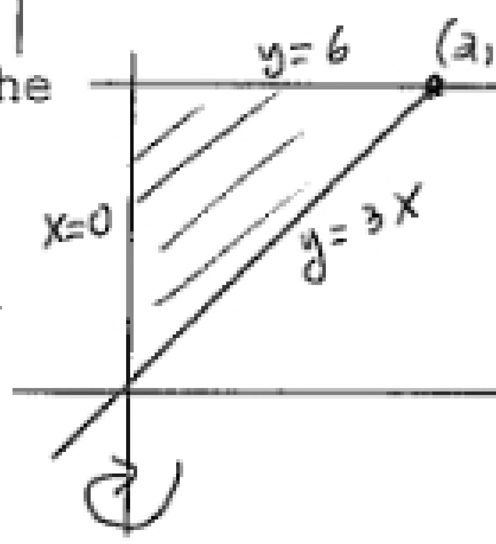
A) $\frac{1}{2}$ B) $\frac{3}{2}$ C) $\frac{5}{2}$ D) $\frac{7}{2}$ E) $\frac{9}{2}$ F) $\frac{11}{2}$ G) $\frac{13}{2}$ H) $\frac{9}{4}$ I) $\frac{17}{4}$ J) $\frac{19}{4}$



Solutions: $\int_0^1 x - x^3\, dx + \int_1^2 x^3 - x\, dx = (\frac{x^2}{2} - \frac{x^4}{4})\big|_0^1 + (\frac{x^4}{4} - \frac{x^2}{2})\big|_1^2 =$
$\frac{1}{2} - \frac{1}{4} + 4 - 2 - \frac{1}{4} + \frac{1}{2} = \frac{5}{2}$ (C)

11. Find the volume of the solid obtained by rotating the region bounded by the lines, $y = 3x$, $y = 6$ and $x = 0$ about the y-axis.

A) 2π B) $\frac{8}{3}\pi$ C) 4π D) $\frac{13}{2}\pi$ E) 5π F) $\frac{17}{3}\pi$ G) 6π H) $\frac{20}{3}\pi$ I) 8π J) $\frac{27}{2}\pi$



Solution: (washer method) $V = \pi\int_0^6 \frac{1}{9}y^2\, dy = \frac{\pi}{27}y^3\big|_0^6 = 8\pi$. (I)

12. Find the **volume** of the **solid** whose **base** is the region in the x-y plane bounded by the circle $x^2 + y^2 = 9$ and whose **cross-sections** perpendicular to the x-axis are **squares**, with one side of the square on the x-y plane.

A) 120 B) 144 C) 162 D) 184 E) 206 F) 220 G) 246 H) 260 I) 284

Solution: $A(x) = (2\sqrt{9 - x^2}\,)^2 = 36 - 4x^2$, $-3 \leq x \leq 3$. Then volume is
$\int_{-3}^{3} 36 - 4x^2\, dx = 2\int_0^3 36 - 4x^2\, dx = 2(36x - \frac{4}{3}x^3)\big|_0^3 = 144$. (B)