EE 380 **EXAM I** **10 October 2012**

Last Name (Print): Solutions

First Name (Print): ____________________

ID number (Last 4 digits): ____________

Section: ___

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Problem	Weight	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

INSTRUCTIONS

1. You have 2 hours to complete this exam.
2. This is a closed book exam. You may use one 8.5" × 11" note sheet.
3. Calculators are allowed.
4. Solve each part of the problem in the space following the question. If you need more space, continue your solution on the reverse side labeling the page with the question number; for example, **Problem 1.2 Continued**. **NO** credit will be given to solutions that do not meet this requirement.
5. **DO NOT REMOVE ANY PAGES FROM THIS EXAM.** Loose papers will not be accepted and a grade of **ZERO** will be assigned.
6. The quality of your analysis and evaluation is as important as your answers. Your reasoning must be precise and clear; your complete English sentences should convey what you are doing. **To receive credit, you must show your work.**

Problem 1: (25 Points)

1. (12 points) Figure 1 shows the block diagram of a feedback control system. The state-space model

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & 1 \\ 0 & -5 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u &= Ax + Bu \\ y &= \begin{pmatrix} 1 & 0 \end{pmatrix} x &= Cx \end{aligned}$$

represents the plant dynamics, while the ODE representation

$$\ddot{u} + 10\,\dot{u} + 50\,u = 50\,m(t)$$

represents the actuator dynamics. Using the proportional control design

$$m(t) = K_P\, e(t),$$

where K_P is the proportional control gain, determine the transfer function $Y(s)/R(s)$ of the closed-loop system, and place your answer in the standard form

$$\frac{Y(s)}{R(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}.$$

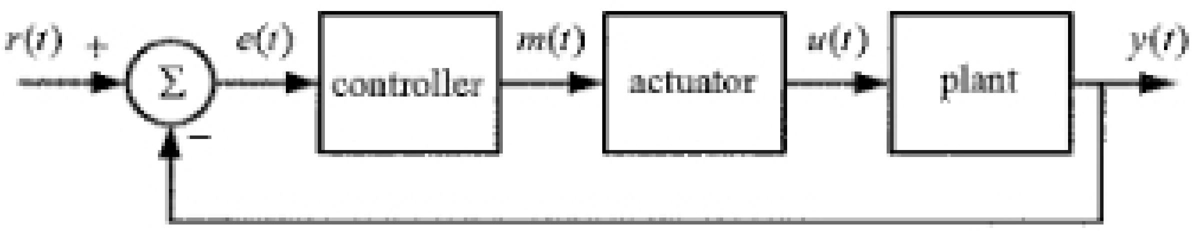

Figure 1: Block diagram of a feedback control system.

Plant transfer function: $\frac{Y}{U} = C(sI-A)^{-1}B = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} s & -1 \\ 0 & s+5 \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\frac{Y}{u} = \frac{1}{s(s+5)}\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} s+5 & 1 \\ 0 & s \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{s(s+5)}\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ s \end{pmatrix} = \frac{1}{s(s+5)}$$

Actuator Dynamics: $\ddot{u} + 10\dot{u} + 50u = 50m(t)$

Actuator Transfer function: $(s^2 + 10s + 50)u = 50M$

$$\frac{U(s)}{M(s)} = \frac{50}{s^2+10s+50}$$

Closed-Loop transfer function:

$$\frac{Y(s)}{R(s)} = \frac{K_p \frac{50}{s^2+10s+50}\frac{1}{s(s+5)}}{1 + K_p\frac{50}{s^2+10s+50}\frac{1}{s(s+5)}} = \frac{50K_p}{s(s+5)(s^2+10s+50) + 50K_p}$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{50K_p}{s^4 + 15s^3 + 100s^2 + 250s + 50K_p}}$$