AAE 340 – Dynamics and Vibrations

Exam II

Solutions

Please read the problems carefully.
Write clearly and use diagrams when necessary.

(55 points)

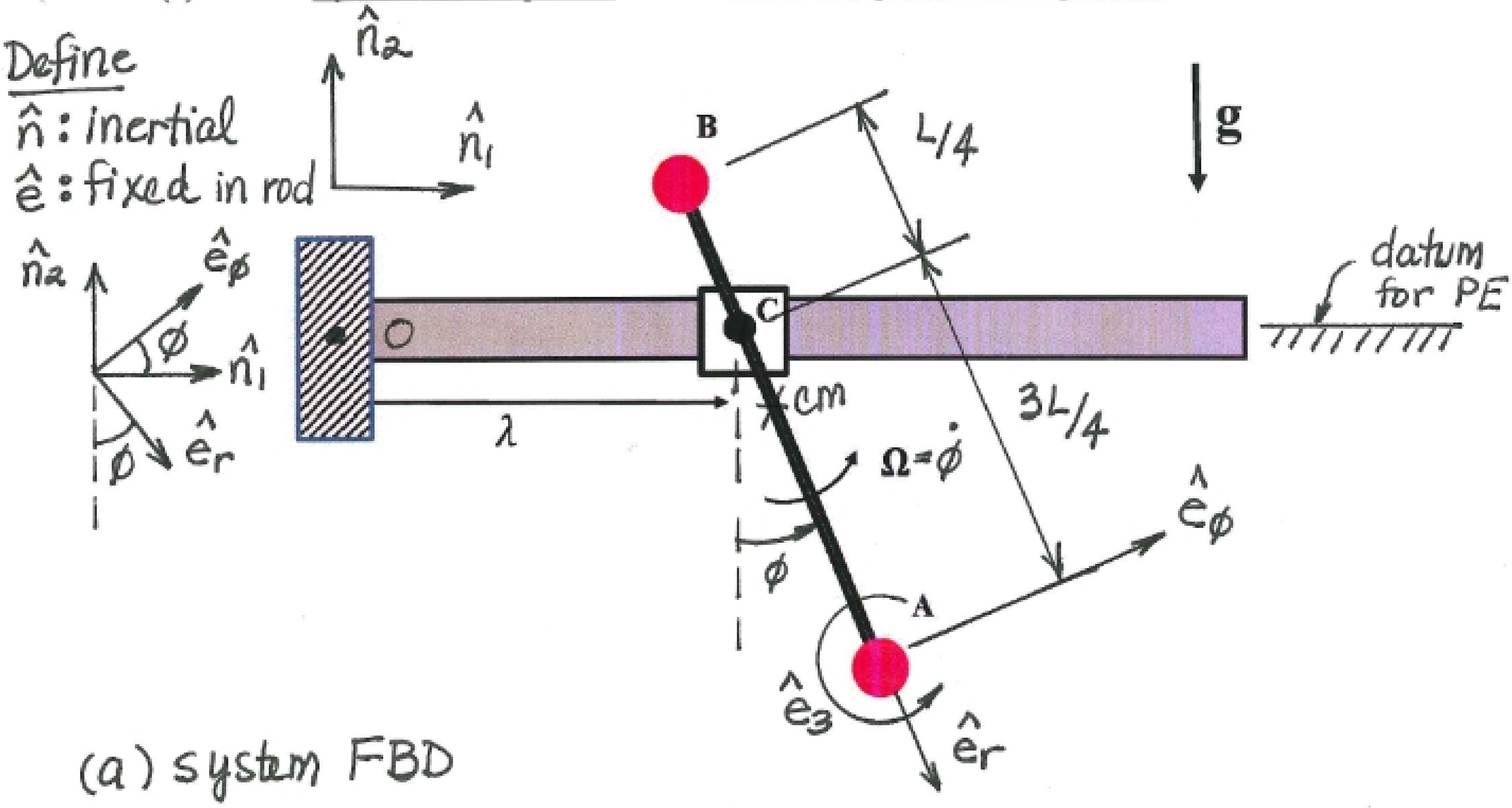
1. The system below is comprised of the two red particles, each of mass m. Massless block C can slide along a frictionless horizontal track. Particle B is connected to A by a massless, inextensible rod of length L; the rod can rotate in a vertical plane at the variable rate Ω. Particle B is located at a distance $L/4$ from C.

   (a) Sketch a system FBD. How many EOM are required and why?

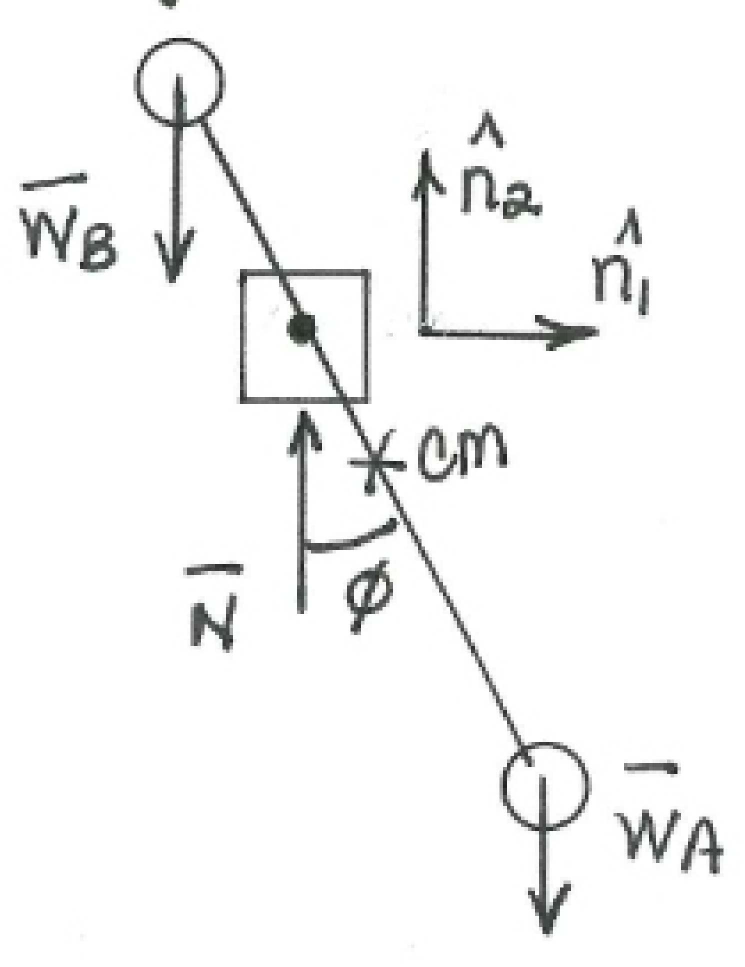
   (b) (i) Total system energy is constant. Justify this statement and write any potential energy expressions if appropriate.
   (ii) Linear momentum in one direction IS conserved. Justify this statement.

   (c) Use the system force equation and the moment equation about point C to derive EOM.

Define
$\hat{n}$: inertial
$\hat{e}$: fixed in rod



(a) system FBD



force models $\bar{W}_B = \bar{W}_A = -mg\,\hat{n}_2$
$\bar{N}$ unknown normal

2 VoI : λ, ϕ

$\Longrightarrow$ 2 EOM

(b) (i) E constant

weight forces are conservative
both weight forces do work but potential energy expressions are available

$$V_B = + mg \frac{L}{4} c\phi \qquad V_A = - mg \frac{3L}{4} c\phi$$

normal force does no work $\underbrace{\bar{N} \cdot {}^{n}\bar{V}^{c}}_{N\hat{n}_2 \cdot \dot{x}\hat{n}_1 = 0} = 0$

internal force in rod does no work because the distance between the particles is <u>fixed</u>

(ii) $\bar{P}_{total}$ is NOT conserved because total external forces do not equal zero $\bar{F}_{external} \neq \bar{0}$
But in inertial direction $\hat{n}_1$:

$$\underbrace{\bar{F}}_{external} \cdot \hat{n}_1 = 0 \longrightarrow \text{so } \bar{p} \cdot \hat{n}_1 \text{ is constant}$$

(c) $\bar{F} = m\, {}^{n}\bar{A}^{cm}$ (${}^{n}\bar{A}^{cm}$ is generic!)

$$\bar{r}^{ocm} = \lambda \hat{n}_1 + \frac{L}{4} \hat{e}_r$$

$${}^{n}\bar{v}^{cm} = \dot{\lambda} \hat{n}_1 + \frac{L}{4} \Omega \hat{e}_\phi$$

$${}^{n}\bar{A}^{cm} = \ddot{\lambda} \hat{n}_1 + \frac{L}{4} \dot{\Omega} \hat{e}_\phi - \frac{L}{4} \Omega^2 \hat{e}_r$$

$${}^{n}\bar{A}^{cm} = \left[\ddot{\lambda} + \frac{L}{4} \dot{\Omega} c\phi - \frac{L}{4} \Omega^2 s\phi\right] \hat{n}_1 + \left[\frac{L}{4} \dot{\Omega} s\phi + \frac{L}{4} \Omega^2 c\phi\right] \hat{n}_2$$

$$\hat{n}_1 : \boxed{\ddot{\lambda} + \frac{L}{4} \dot{\Omega} c\phi - \frac{L}{4} \Omega^2 s\phi = 0} \quad \text{EOM}$$

$$\hat{n}_2 : m\left[\frac{L}{4} \dot{\Omega} s\phi + \frac{L}{4} \Omega^2 c\phi\right] = N - 2mg$$