

PROBLEM 1

Given Eqn to model effect of air resistance on a vehicle

$$y = \frac{bu + x}{\beta^3} (1 - ce^{-ux}) - \frac{\gamma x}{u} + e$$

where $\dim[e] = L$


$$\dim[u] = 1/T$$

Find The dimensions (not units!) of each quantity

Assume: e^{-ux} is an exponential

Solution I'm adopting the International System of Units (SI) that states as fundamental quantities

- 1) Mass M
- 2) Length L
- 3) Time T

* For the equation to be dimensionally correct, all the right hand side (RHS) terms must have the same dimension 

$$\dim \left[\frac{bu + x}{\beta^3} \right] = \dim \left[\frac{bu + x}{\beta^3} ce^{-ux} \right] = \dim \left[\frac{\gamma x}{u} \right] = \dim[e]$$

with $\dim[e] = L$ Known!

Also, quantities can be added only if they have the same dimension.

Thus

$$\dim[1] = \dim[ce^{-vz}]$$

$$1 = \dim[c] \dim[e^{-vz}] \rightarrow \underline{\dim[c] = 1} *$$

$\downarrow = 1!$

Also, the power $-vz$ of e^{-vz} itself dimensionless

$$\dim[vz] = \dim[v] \dim[z] = \frac{1}{T} \dim[z] = 1$$

$$\hookrightarrow \underline{\dim[z] = T} *$$

Eventually

$$\bullet \dim\left[\frac{zx}{v}\right] = \dim[e]$$

$$\frac{\dim[z] \dim[x]}{\dim[v]} = L$$

$$T^2 \dim[x] = L \rightarrow \underline{\dim[x] = L/T^2} *$$

$$\bullet \dim[bv] = \dim[x]$$

$$\dim[b] \frac{1}{T} = \frac{L}{T^2} \rightarrow \underline{\dim[b] = \frac{L}{T}} *$$

$$\bullet \dim \left[\frac{x}{\beta^3} \right] = \dim [e]$$

$$\dim [x] \frac{1}{\dim[\beta]^3} = L$$

$$\frac{L}{T^2} \frac{1}{\dim[\beta]^3} = L \rightarrow \dim[\beta] = \sqrt[3]{\frac{L}{T^2}} \quad *$$

Finally, the LEFT hand side (LHS) of the eqn must have the same dimension of the RHS

$$\dim [y] = \dim [\text{RHS}] = \dim [e] = L \quad *$$

PROBLEM 2

Given $2\ddot{p} - \eta \ddot{a} c_a + \eta \dot{a}^2 s_a - 2g s_{30} = -N [2g c_{30} - \eta \ddot{a} s_a + \eta \dot{a}^2 c_a]$

where $\ddot{p} = \frac{d^2 p}{dt^2}$ $\dot{p} = \frac{dp}{dt}$

$$\ddot{a} = \frac{d^2 a}{dt^2} \quad \dot{a} = \frac{da}{dt}$$

$$s_a = \sin a \quad c_a = \cos a$$

Find Dimensions of p, η, a, N . Eqn. dimly consistent?