AAE 340 – Dynamics and Vibrations
Problem Set 7
Due: 10/23/13

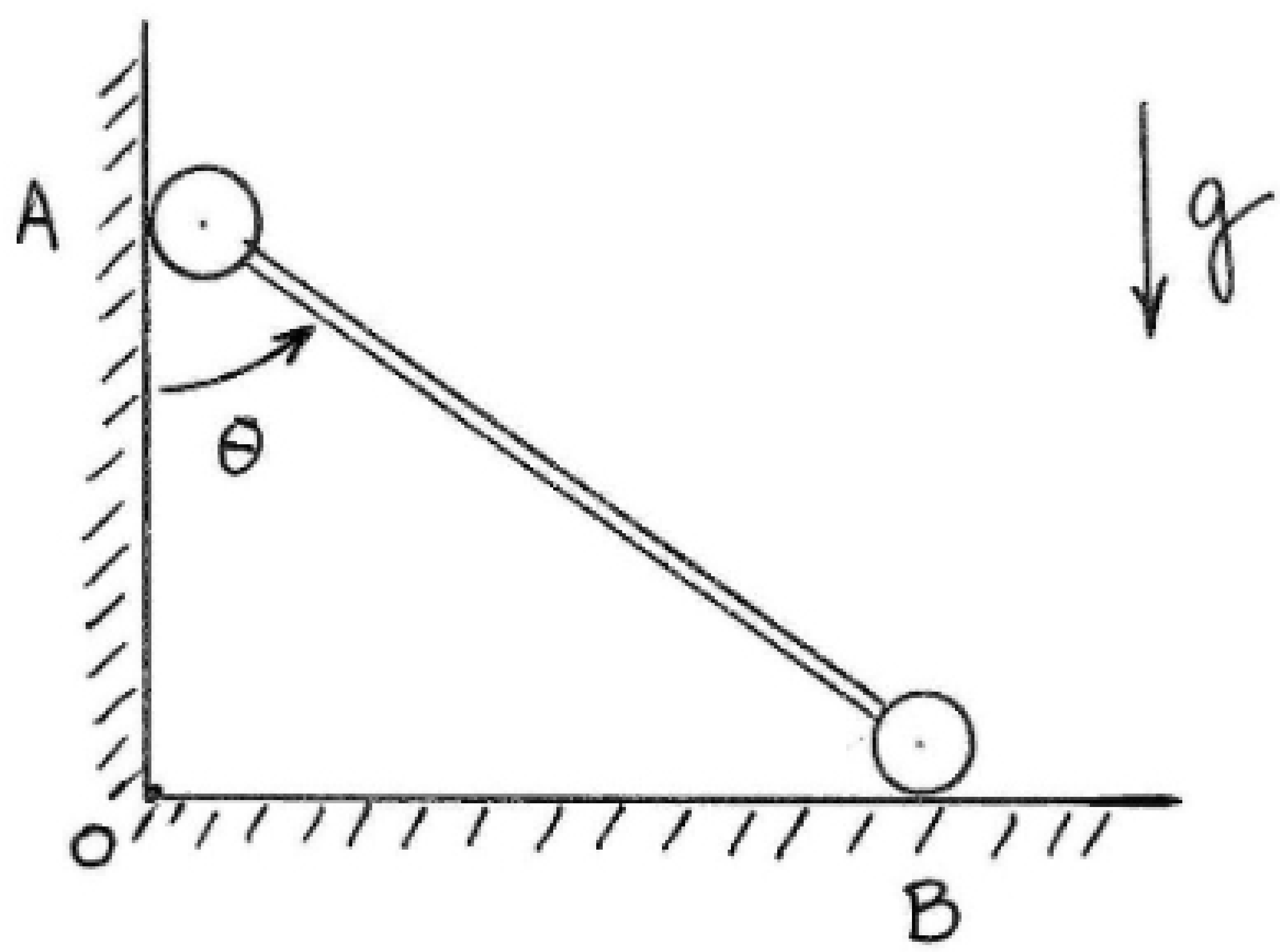
Problem 1: A dumbbell consists of two particles, each of mass m, connected by a rigid massless rod of length L as indicated in the figure. The upper end of the dumbbell, located at point A, slides without friction along a vertical wall, while the lower end of the dumbbell, located at point B, slides without friction along a horizontal floor. Let θ be the angle between the vertical wall and the rod. Assume the initial conditions are $\theta(0)=0$ and $\dot{\theta}(0)=0$.

(a) Use the force equation for a system and the moment equation about the center of mass and derive the differential equation that governs motion while the dumbbell maintains contact with the wall and the floor.

(b) Total system energy is constant. Justify that statement and write an expression for E.

(c) Determine the angle $\theta=\theta^*$ at which the dumbbell loses contact with the vertical wall.

(d) Let the mass of each particle be 2 kg and the length of the rod is 25 ft. Numerically integrate the EOM till the rod loses contact with the wall. How long till the condition is met? Plot $\theta(t)$ and the normal forces at the contact points as functions of time.



Problem 2: A system consists of two blocks 1 and 2 of masses m_1 and m_2, respectively. The blocks slide without friction inside a circular slot of radius R cut from a disk of radius R. Furthermore, a curvilinear spring of spring constant K and unstretched length ℓ_o is attached between the two blocks. Let θ_1 describe the position of block 1 while θ_2 describe the position of block. Then, $\hat{n}_i$ is defined as an inertial set of unit vectors.

(a) Define all the appropriate quantities and sets of unit vectors. Define the VOI; how many variables of interest are required in the problem? Two EOMs are necessary to describe the motion of the system. Why?

(b) Derive an expression for ${}^{n}\bar{A}^{cm}$. Is the center of mass located along the spring?

(c) Sketch Three FBDs: the system FBD; a FBD for block 1; a FBD for block 2.
Consider your options:

(i) If you use the system FBD and $\bar{M}^{block1}$, how many EOM will emerge? Why?

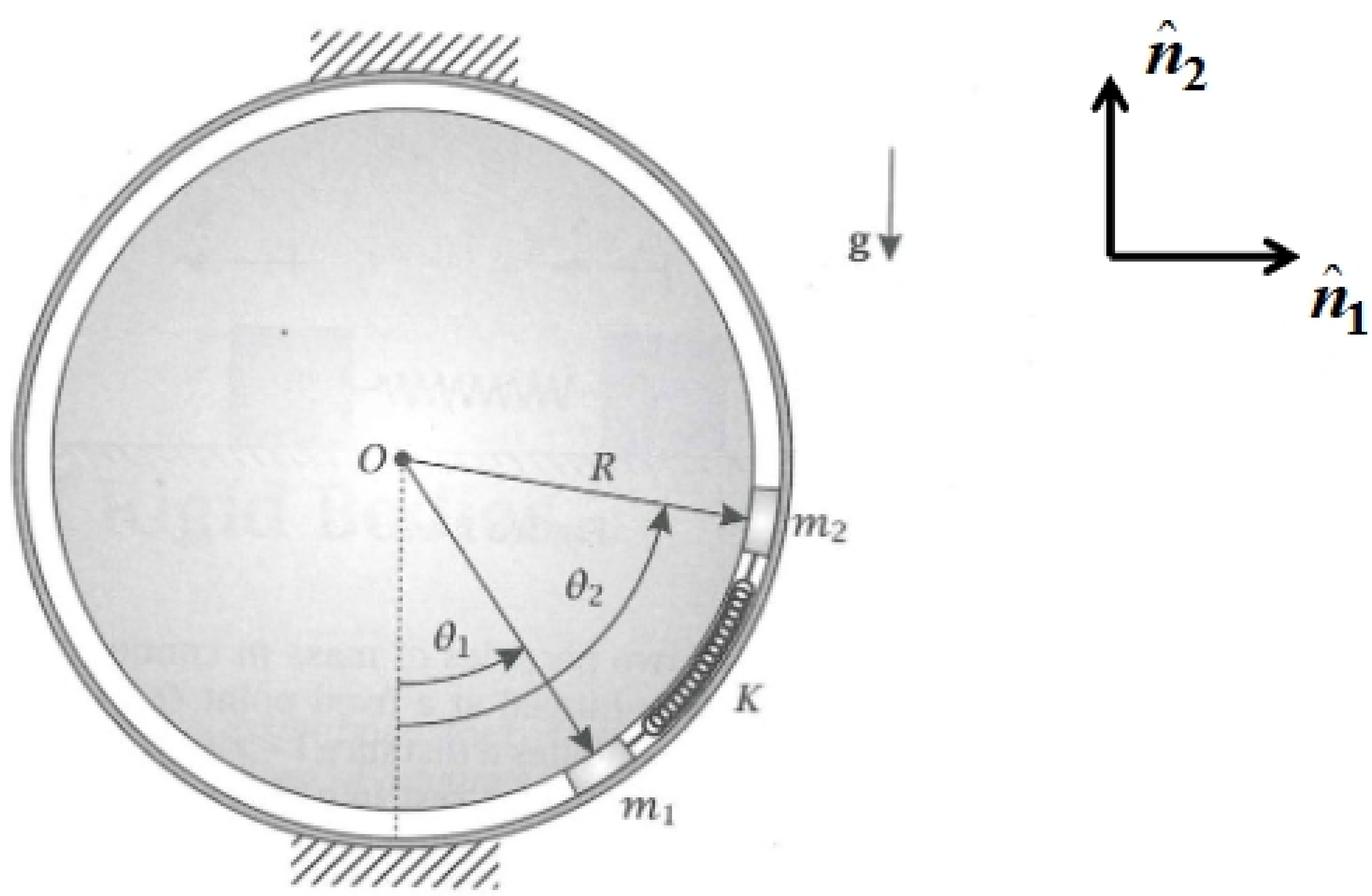
(ii) If you use the system FBD and $\bar{M}^{O}$, how many EOMs are available? Why?

(iii)If you use the FBD of block 1, how many EOMs will be generated? Why?

(iv)If you use the system FBD and the $\bar{F}$ equation, how many EMOs can you derive? Why?

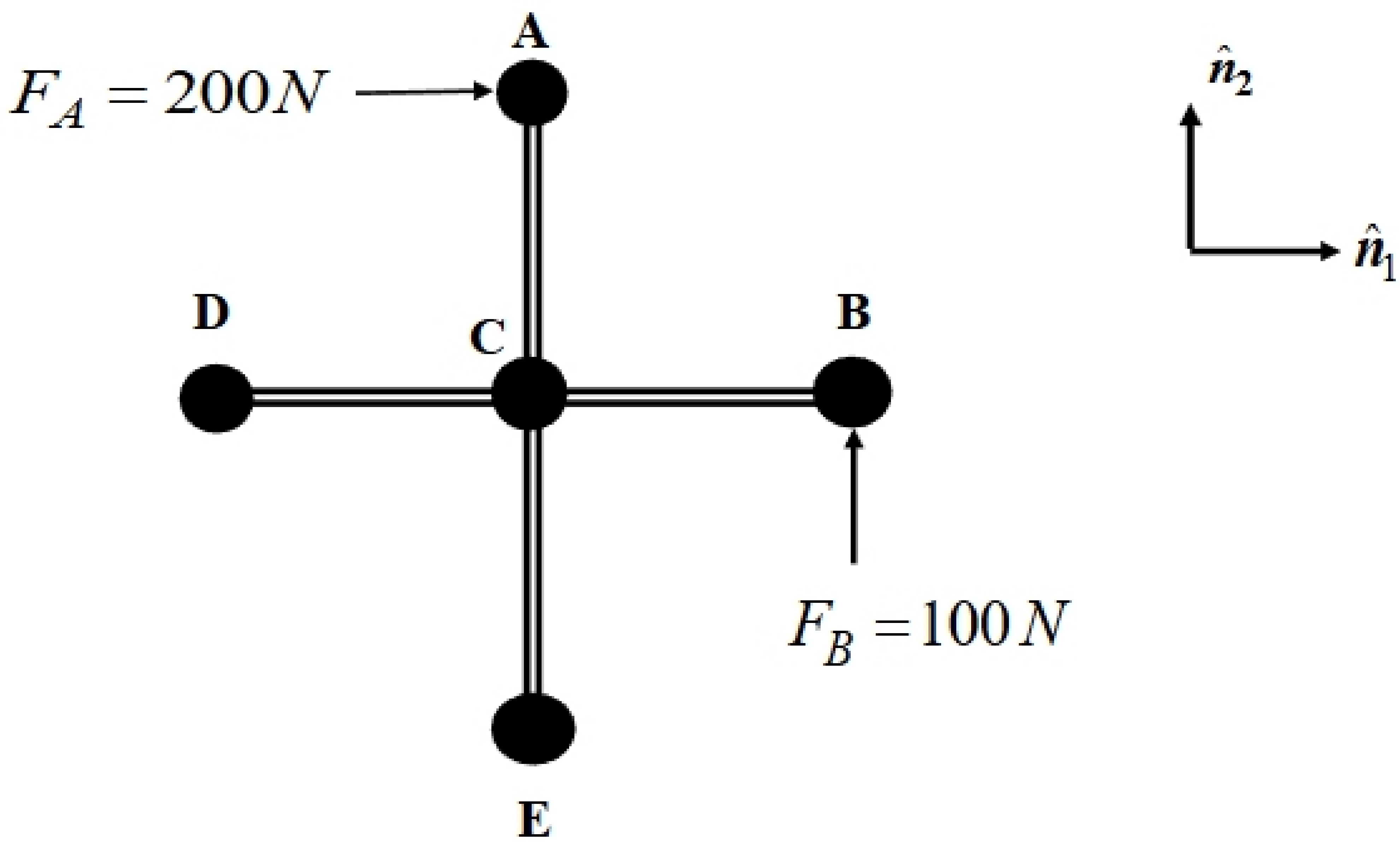
What is the best combination to generate the EOMs with the least number of steps?
Use your selected combination and derive the EOM.

(d) Derive an expression for the total system energy. Are all the forces conservative? Justify that answer.



Problem 3: A system consists of five identical particles connected by massless inextensible rigid rods. The system can move on a smooth, frictionless horizontal surface. Initially, the system is at rest. Unit vectors $\hat{n}_i$ are defined as an inertial set. The particles are each 5-lb and the rods are 10-ft long between each pair of particles.

(a) At $t = 0$, two forces are applied to particles A and B as indicated in the given inertial directions.



If the external forces are constant and act in their respective inertial directions for 2 seconds, deteremine the net linear impulse that acts on the system.

At $t = 2\,\text{s}$, use linear impulse and linear momentum and determine the velocity of the center of mass.

(a) Assume that the forces are removed. Dereive an expression for the total kinetic energy of this system.

At some future time, the ${}^{n}\bar{v}^{cm} = 4\hat{n}_1 - 3\hat{n}_2$ ft/s and the system is rotating on the horizontal surface at the rate ${}^{n}\bar{\omega}^{b} = 25\hat{n}_3$ rpm. Evaluate the total system kinetic energy at that time.