AAE 340
Problem Set 6

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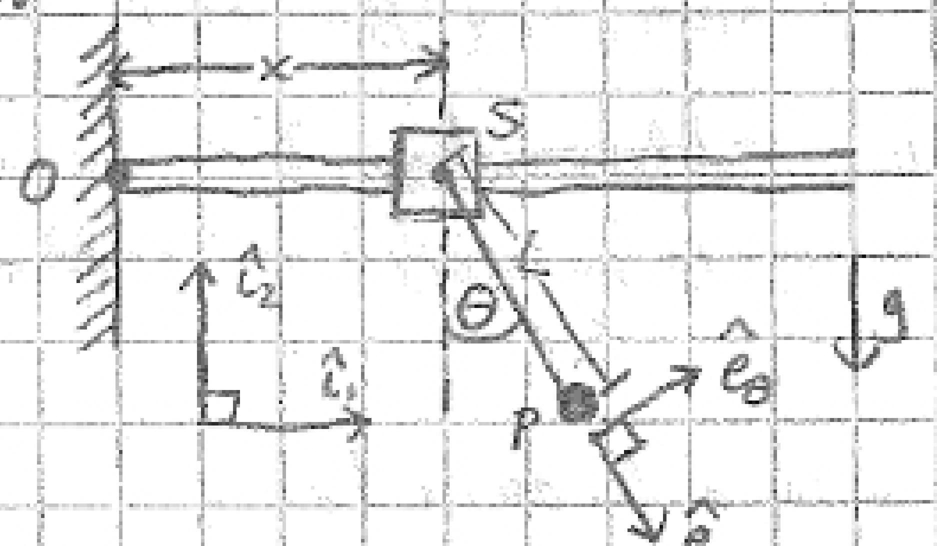
Problem 1)

Given: - Particle P with mass m attached to rigid rod
- Rod rotates in the vertical plane
- Rod attached to massless slider on frictionless track

Find: (a) Appropriate quantities; constraints
(b) EOM from $\bar{F} = m\,{}^i\bar{A}^P$
(c) EOM from $\bar{M}^O = \frac{{}^i d\,{}^i\bar{H}^O}{dt}$
(d) Constant component of linear momentum
Relation to EOM
Constant energy? Constant angular momentum?
(e) $\dot{x}$ and $\dot{\theta}$ when $\theta = 0^\circ$ given
$m = 2\,kg,\ L = 6\,m,\ \theta(0) = 60^\circ,\ \dot{\theta}(0) = 0,\ x(0) = 1\,m,\ \dot{x}(0) = 0$

Diagram:



$\hat{\imath}$: Inertial
$\hat{e}$: Body, in the frame of rod

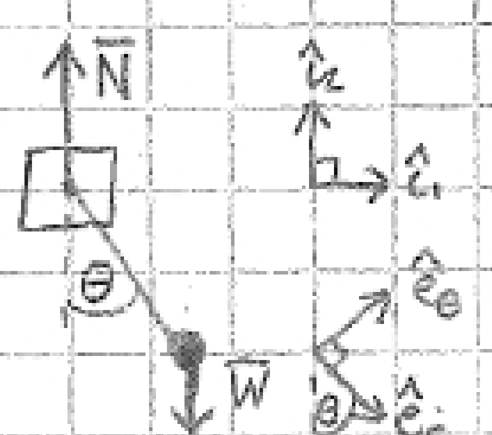
$${}^i\bar{\omega}^e = \dot{\theta}\,\hat{\imath}_3 = \dot{\theta}\,\hat{e}_3 \text{ (out of page)}$$

Rotation matrix:

	$\hat{e}_\theta$	$\hat{e}_r$
$\hat{\imath}_1$	c_θ	s_θ
$\hat{\imath}_2$	s_θ	$-c_\theta$

Length of rod: L; Variables of interest: x, θ

FBD:



Force model:

Normal: $\bar{N} = N\,\hat{\imath}_2$

Weight: $\bar{W} = -mg\,\hat{\imath}_2$

$$\bar{F} = \bar{N} + \bar{W} = (N - mg)\,\hat{\imath}_2$$

(a) $\overline{r}^{OS} = x\hat{\imath}_1$, $\overline{r}^{SP} = L\hat{e}_r \rightarrow \overline{r}^{OP} = x\hat{\imath}_1 + L\hat{e}_r$

Since $\hat{e}_r = s_\theta\hat{\imath}_1 - c_\theta\hat{\imath}_2$, $\overline{r}^{OP} = (x + L\sin\theta)\hat{\imath}_1 - L\cos\theta\,\hat{\imath}_2$

$${}^i\overline{V}^{OP} = \dot{\overline{r}}^{OP} = (\dot{x} + L\dot{\theta}\cos\theta)\hat{\imath}_1 + (L\dot{\theta}\sin\theta)\hat{\imath}_2$$

$${}^i\overline{A}^{OP} = {}^i\dot{\overline{V}}^{OP} = (\ddot{x} + L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta)\hat{\imath}_1 + (L\ddot{\theta}\sin\theta + L\dot{\theta}^2\cos\theta)\hat{\imath}_2$$

The fact that the rod is inextensible means the mass is always a distance L from the slider. We can then treat the rod and mass together, giving us the two VOI, x and θ. Noting the constraining normal force N, we can solve for those VOI, since this restricts the slider's motion to one dimension.

(b) $\overline{F} = m\,{}^i\overline{A}^P$

As previously found, $\overline{F} = (N - mg)\hat{\imath}_2$, and ${}^i\overline{A}^P = (\ddot{x} + L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta)\hat{\imath}_1 + (L\ddot{\theta}\sin\theta + L\dot{\theta}^2\cos\theta)\hat{\imath}_2$

So, $(N - mg)\hat{\imath}_2 = m(\ddot{x} + L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta)\hat{\imath}_1 + m(L\ddot{\theta}\sin\theta + L\dot{\theta}^2\cos\theta)\hat{\imath}_2$

This gives us two equations from the $\hat{\imath}_1$ and $\hat{\imath}_2$ directions. There are no components in $\hat{\imath}_3$

$$0 = m(\ddot{x} + L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta), \quad N - mg = m(L\ddot{\theta}\sin\theta + L\dot{\theta}^2\cos\theta)$$

The second equation gives our definition of the normal force

$$N = mg + mL\ddot{\theta}\sin\theta + mL\dot{\theta}^2\cos\theta$$

The first equation is our equation of motion:

$$\boxed{\ddot{x} + L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta = 0}$$

(c) $\bar{M}^O = \frac{{}^i d\, {}^i\bar{H}^O}{dt}$

$\bar{M}^O = \sum \bar{r}\times\bar{F} \rightarrow \bar{M}^O = (\bar{r}^{OS}\times\bar{N}) + (\bar{r}^{OP}\times\bar{W})$

$= [(x\hat{\imath}_1)\times(N\hat{\imath}_2)] + [((x+L\sin\theta)\hat{\imath}_1 - L\cos\theta\,\hat{\imath}_2)\times(-mg\hat{\imath}_2)]$

$= [Nx\,\hat{\imath}_3] + [-mg(x+L\sin\theta)\hat{\imath}_3 + 0]$

$= (Nx - mgx - mgL\sin\theta)\hat{\imath}_3$

${}^i\bar{H}^O = \bar{r}^{OP}\times m\,{}^i\bar{v}^P$

${}^i\bar{H}^O = [(x+L\sin\theta)\hat{\imath}_1 - (L\cos\theta)\hat{\imath}_2]\times m[(\dot{x}+L\dot\theta\cos\theta)\hat{\imath}_1 + (L\dot\theta\sin\theta)\hat{\imath}_2]$

$= m(x+L\sin\theta)(L\dot\theta\sin\theta)\hat{\imath}_3 - m(L\cos\theta)(\dot x + L\dot\theta\cos\theta)(-\hat{\imath}_3)$

$= (mxL\dot\theta\sin\theta + \underline{mL^2\dot\theta\sin^2\theta} + m\dot x L\cos\theta + \underline{mL^2\dot\theta\cos^2\theta})\hat{\imath}_3$

$= (mxL\dot\theta\sin\theta + m\dot xL\cos\theta + mL^2\dot\theta)\hat{\imath}_3$

$\frac{{}^i d\,{}^i\bar{H}^O}{dt} = [(\cancel{m\dot xL\dot\theta\sin\theta} + mxL\ddot\theta\sin\theta + mxL\dot\theta^2\cos\theta) + (m\ddot xL\cos\theta - \cancel{m\dot xL\dot\theta\sin\theta})$
$+ (mL^2\ddot\theta)]\hat{\imath}_3$

$= (mxL\ddot\theta\sin\theta + mL^2\ddot\theta + mxL\dot\theta^2\cos\theta + m\ddot xL\cos\theta)\hat{\imath}_3$

Therefore, the equation $\frac{{}^i d\,{}^i\bar H^O}{dt} = \bar M^O$ gives

$Nx - mg(x+L\sin\theta) = mxL\ddot\theta\sin\theta + mL^2\ddot\theta + mxL\dot\theta^2\cos\theta + m\ddot xL\cos\theta$

We recall that

$N - mg = m(L\ddot\theta\sin\theta + L\dot\theta^2\cos\theta)$

And note that

$Nx - mgx = (N-mg)x$