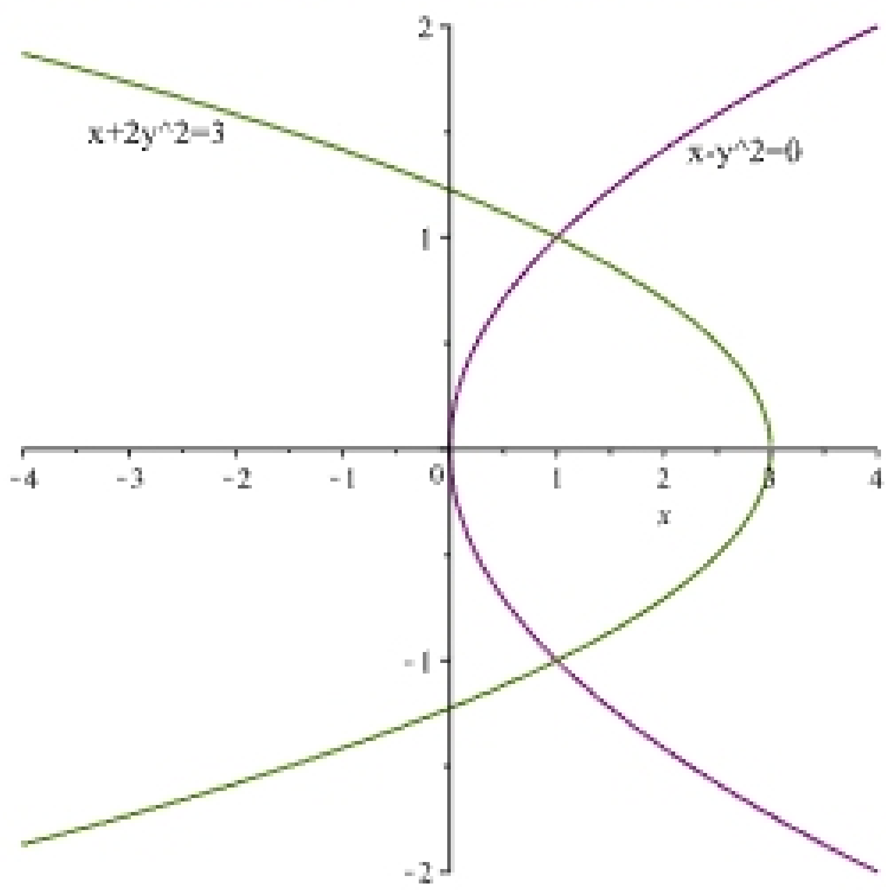
Calculus II PLTL
Fall 2014
Worksheet 1

These problems are to be done without the use of a calculator unless otherwise specified.

1) *(Pairs)* Each of the following definite integrals represents an area that can be computed using facts from geometry. For each integral, sketch the region described and give the area of the region.

$$(a)\quad \int_1^5 4\,dx \quad (b)\quad \int_1^5 3x\,dx \quad (c)\quad \int_1^5 |4-3x|\,dx \quad (d)\quad \int_0^2 \sqrt{4-x^2}\,dx$$

2) *(Round Robin)* Find the area of the region enclosed by the curves pictured below.



3) *(Scribe)* Find the area of the region in the first quadrant bounded on the left by the y-axis and on the right by the curves $y = \sin x$ and $y = \cos x$.

4) *(Round Robin)* Let $f(x) = -x^2$.

(a) Graph f on the interval $[0, 1]$.

(b) Partition the interval into four subintervals of equal length.

(c) Add to your sketch the rectangles associated with the Riemann sum $\sum_{k=1}^{4} f(c_k)\Delta x_k$, where c_k is the left-hand endpoint of the k^{th} interval.

(d) How is the integral $\int_0^1 -x^2\,dx$ related to the limit $\lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\Delta x_k$?

(e) Using what you have learned about forming a Riemann sum using left endpoints, find a function g and numbers a and b such that

$$\int_a^b g(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} g(c_k)\Delta x_k$$

$$= \lim_{n\to\infty} \left((4)^{2/3}\frac{4}{n} + (4+\frac{4}{n})^{2/3}\frac{4}{n} + \cdots + (4+(n-1)\frac{4}{n})^{2/3}\frac{4}{n} \right)$$

5) *(Scribe)* Evaluate $\int_0^8 t\sqrt{t+1}\,dt$.

6) *(Pairs)* Consider the function $g(x) = \int_0^{\cos x} x^2 \cos(2t)\,dt$.

(a) Evaluate the integral to get an alternate expression for $g(x)$, and then find $g'(x)$.

(b) Use the Fundamental Theorem of Calculus (and the Chain Rule) to find $g'(x)$ from the original expression, then compare with your previous answer.