Calculus II PLTL
Fall 2014
Worksheet 9

These problems are to be done without the use of a calculator unless otherwise specified.

1) *(Round Robin)* Use the Ratio Test to explore the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{3^n}$.

2) *(Pairs)* State the conditions under which the Alternating Series Test can be used. Then explain how the test can be used to determine whether a series converges or diverges.

(a) Evaluate the convergence or divergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1}$.

(b) Explain why the Alternating Series Test cannot be used to test $\sum_{n=1}^{\infty}(-1)^n \frac{n^2}{n+1}$ for convergence.

It is a fact that

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}.$$

(c) Show that this series satisfies the requirements of the Alternating Series Test.

If the Alternating Series Test is applicable to a convergent series $\sum a_n$, you can approximate the exact value of the sum to a specified degree of accuracy by taking a large enough partial sum. For example, if the sum of the series, s, is estimated with the partial sum $s_4 = a_0 + a_1 + a_2 + a_3 + a_4$, then the absolute value of the error, $|s - s_4| = |\sum_{n=5}^{\infty} a_n|$ must be less than $|a_5|$.

(d) Explain in your own words why the last statement is true.

(e) Verify that $|s - s_4| \leq |a_5|$ for this series. Note that s_4 estimates the value of e^{-1} correctly to two decimal places.

(f) Now estimate the value of $1/e$ accurate to four decimal places.

3) *(Scribe)* Use the Comparison Test to show that the series converges.

$$\sum_{n=1}^{\infty} \left(\frac{n^2}{3n^2+4}\right)^n$$

4) *(Pairs)* Determine whether each of the following series is *absolutely convergent, conditionally convergent*, or *divergent.*

$$(a) \sum_{n=3}^{\infty} \frac{(-1)^n}{\ln n} \quad (b) \sum_{n=0}^{\infty} \frac{(-1)^n n^3}{2^n}$$

5) *(Round Robin)* Determine whether each series is convergent or divergent. Make sure to indicate which tests can be used, and whether any tests are indeterminate for each series.

(a) $\sum_{n=2}^{\infty}(n^2-1)/(n^3-n-1)$

(b) $\sum_{n=2}^{\infty} 1/(n\sqrt{\ln n})$

(c) $\sum_{n=0}^{\infty} \frac{3^n}{2^n+4^n}$

(d) $\sum_{n=1}^{\infty} 1000/\sqrt{n^3+2}$

(e) $\sum_{n=0}^{\infty} \sin n$

6) *(   )* Use an example or a picture to illustrate whether each statement is true or false.

(a) For $|r|<1$ and a a real number, $\sum_{n=1}^{\infty} ar^{n-1}$ has the same sum as $\sum_{n=0}^{\infty} ar^n$.

(b) If a series $\sum a_n$ is such that the terms a_n tend to zero as n increases, then it is still possible that the series is divergent.

(c) If $\sum |a_n|$ converges, then $\sum(-1)^n|a_n|$ converges.

(d) If the sequence a_n is not bounded, and a_n is positive for $n > 100$, then the sequence has a term greater than 1,000,000.

(e) If $f(x) > 0$ and f is decreasing and $\int_a^{\infty} f(x)\,dx = K < \infty$ for some real number a, then $\sum_{n=1}^{\infty} f(n)$ is convergent.

(f) If a monotonic sequence of positive terms does not converge, then it has a term greater than one googol.