

Calculus II PLTL

Fall 2014

Worksheet 11

**These problems are to be done without the use of a calculator unless otherwise specified.**

1) (*Scribe*) Let  $f(x) = x^3 + x^2 + x + 1$ .

(a) Find the Taylor series of  $f$  at  $a = 1$ .

(b) What expression will you get if you multiply out all of the terms of the Taylor series? Explain.

2) (*Round Robin*) Let  $f(x) = \ln(1 - 2x) - \tan^{-1}(x/2)$ .

(a) Use the definition of a Taylor Series to find the coefficients of the terms of degree less than five of the Taylor series at  $x = 0$  for the function.

(b) Use the series expression for  $\ln(1 + x)$  and  $\arctan(x)$  to check your answer.

(c) What is the interval of convergence for this series?

**3)** (*Pairs*) Let  $f$  have a power series representation  $T$ , and suppose that

(i)  $f(0) = 1$

(ii)  $f'(0) = 1$

(iii)  $f''(0) = 2$

(iv)  $f^{(3)}(0) = 6$

(a) If the above is the only information we have about  $f$ , to what degree of accuracy can we estimate  $f(1)$ ? Explain.

(b) If, in addition, we know that  $T$  converges on the interval  $[-2, 2]$  and that  $|f^{(4)}(x)| \leq 15$  on that interval, then to what degree of accuracy can we estimate  $f(1)$ ?

**4)** (*Round Robin*) Find the radius of convergence and the interval of convergence of the series.

(a)  $\sum_{n=1}^{\infty} 5^n x^n$

(b)  $\sum_{n=1}^{\infty} x^n / n^5$

(c)  $\sum_{n=1}^{\infty} n^n x^n$

(d)  $\sum_{n=1}^{\infty} x^n / n^n$

**5)** (*Scribe*) The Maclaurin series for  $e^x$  is  $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ , which is convergent for all  $x$ .

(a) Give the Maclaurin series for  $3e^x$  using summation notation.

(b) Give the Maclaurin series for  $e^{-3x}$  using summation notation.

(c) Give the Maclaurin series for  $e^x + 3$  using summation notation.

(d) Give the Maclaurin series for  $e^{x+3}$  using summation notation.