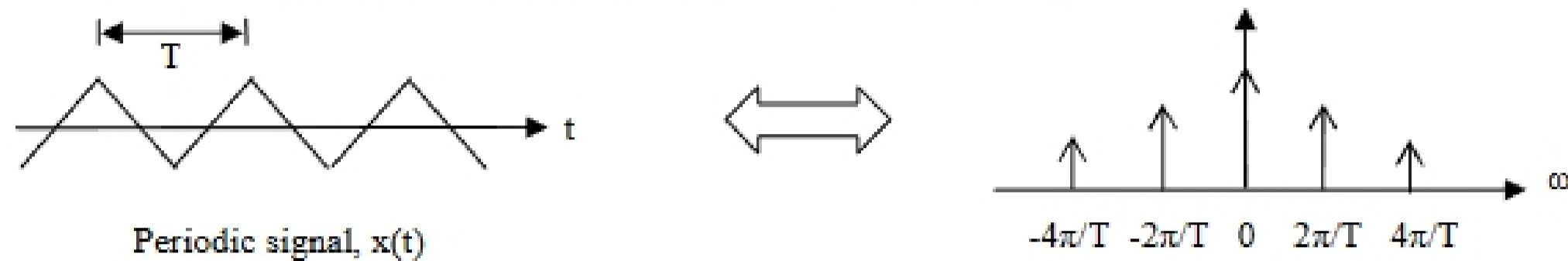


## Chapter 10: Discrete Fourier Transform & Fast Fourier Transform

An assortment of “Fourier” analysis methods:

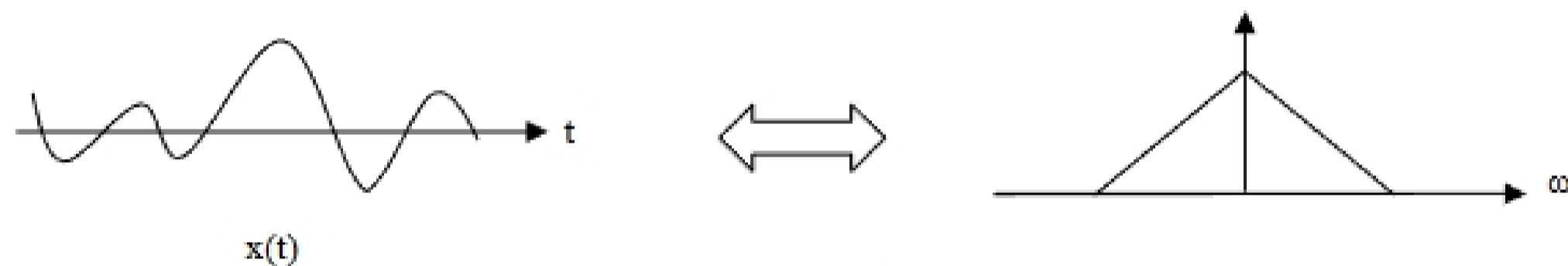
### 1. Fourier Series – continuous-time periodic signals



$$\text{Analysis : } X_k = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-jk\frac{2\pi}{T}t} dt$$

$$\text{Synthesis : } x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\frac{2\pi}{T}t}$$

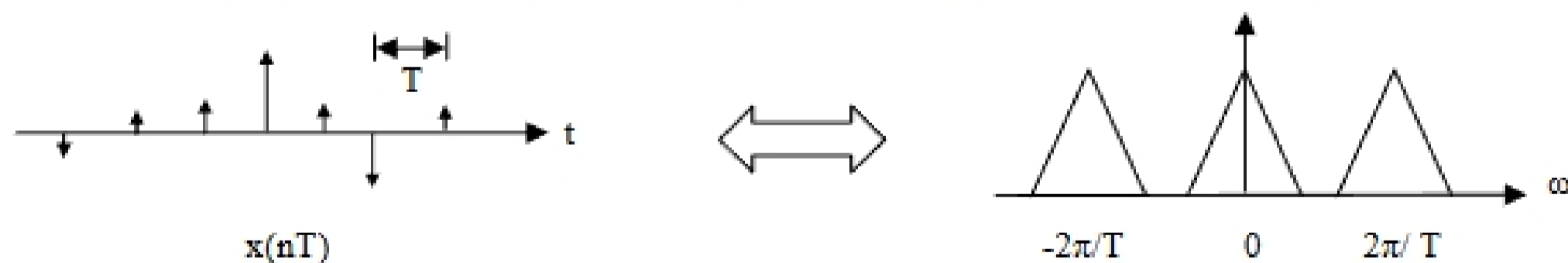
### 2. Fourier Transform – general continuous-time signals



$$\text{Analysis : } X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$\text{Synthesis : } x(t) = \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

### 3. Discrete-time Fourier Transform – general discrete-time signals

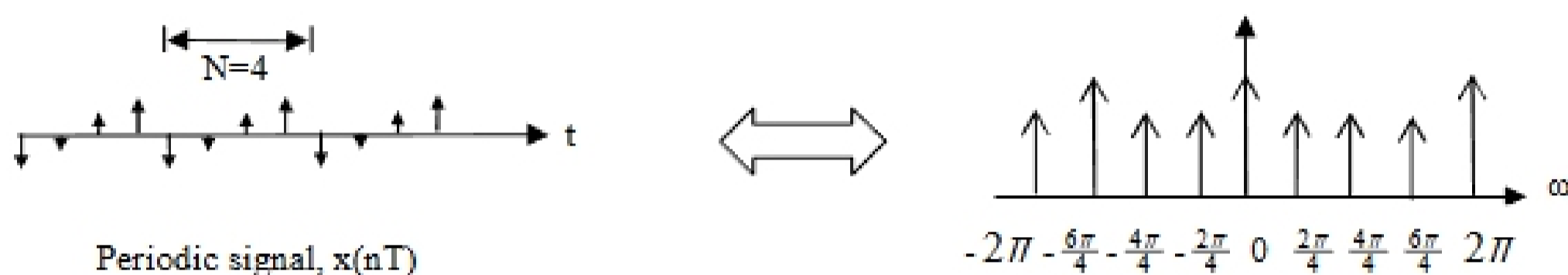


$$\text{Analysis : } X(e^{j\omega T}) = \sum_{n=-\infty}^{\infty} x(nT) e^{-jn\omega T}$$

$$\text{Synthesis : } x(nT) = \int_{-\infty}^{\infty} X(e^{j\omega T}) e^{jn\omega T} d\omega$$

Signals	Fourier	Transform Characteristics
Continuous in t & Periodic	Fourier Series	Discrete in $\omega$
Continuous in t	Continuous-time Fourier Transform	Continuous in $\omega$
Discrete in t	Discrete-time Fourier Transform	Continuous in $\omega$ & Periodic
<b>Discrete in t &amp; Periodic</b>	<b>Discrete Fourier Transform</b>	Discrete in $\omega$ & Periodic

Now we introduce the fourth one, the Discrete Fourier Transform (DFT).

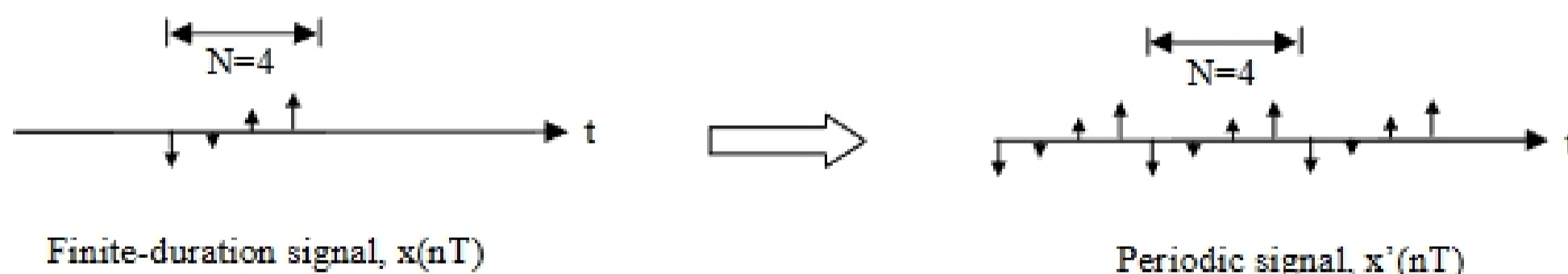


$$\text{Analysis: } X_k = \sum_{n=0}^{N-1} x(nT) e^{-j\frac{2\pi k}{N}n}, \quad k = 0, 1, \dots, N-1$$

$$\text{Synthesis: } x(nT) = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi k}{N}n}, \quad n = 0, 1, \dots, N-1$$

Why DFT? **FOR “APPROXIMATING” DTFT OF A FINITE-DURATION SIGNAL**

In real-life computations, ALL SIGNALS ARE FINITE – the consequence is that you can apply periodic extension to create a periodic discrete-time signal:



What is the relation between the DTFT of the finite duration signal  $x(nT)$  and the DFT of the periodic extended signal  $x'(nT)$ ?

DFT of  $x'(nT)$  are in fact **FREQUENCY SAMPLES** of the DTFT of  $x(nT)$

Here is the definition of *Discrete Fourier Transform (DFT)* again:

$$\text{Analysis: } X_k = \sum_{n=0}^{N-1} x(nT) e^{-j\frac{2\pi k}{N}n}, \quad k = 0, 1, \dots, N-1$$

$$\text{Synthesis: } x(nT) = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi k}{N}n}, \quad n = 0, 1, \dots, N-1$$

Compared with the analysis formula of the *DTFT of a finite-duration*  $x(n)$ :

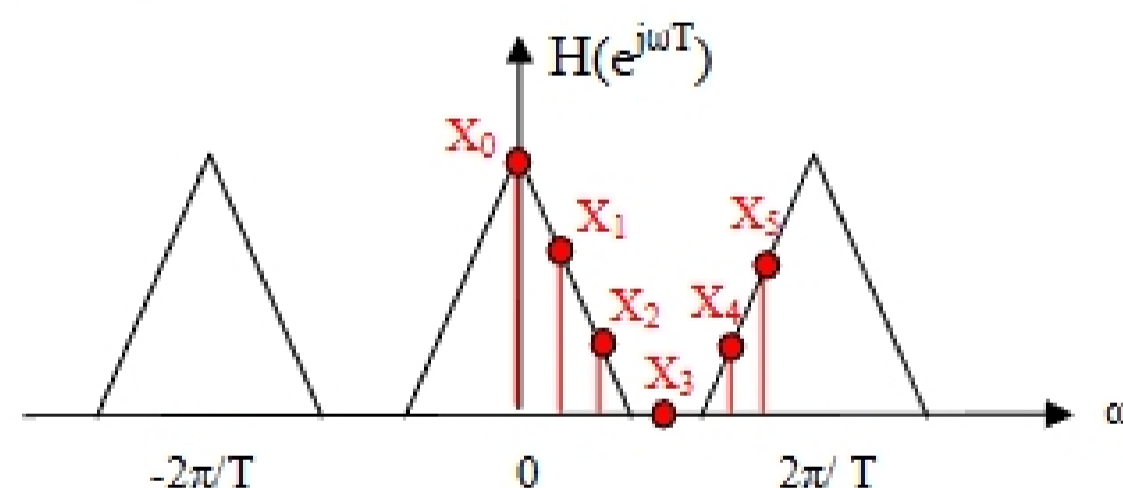
$$X(e^{j\omega T}) = \sum_{n=0}^{N-1} x(nT) e^{-jn\omega T}$$

It is easy to see that  $X_k$  equals  $X(e^{j\omega T})$  evaluated at  $\omega = \frac{2\pi}{T} \frac{k}{N}$

Question 1: What does it mean?

Recall that  $X(e^{j\omega T})$  is periodic with period equal to the sampling frequency  $f_s$  or  $2\pi/T$ , thus  $X_k$  represents the  $k$ -th sample when sampling  $X(e^{j\omega T})$  with  $N$  samples per period.

For example: if  $N=6$ , we have



Question 2: Is there any loss in information (do we need something similar to the Nyquist theorem in Frequency domain)?

**No, as long as the number of samples per period is the same as or larger than the duration of the signal.**

Why?  $X_k$  is also the coefficient for the Discrete-time Fourier series of the periodic signal  $x'(nT)$ .