CDS 101: Lecture 2.1
System Modeling

Richard M. Murray

7 October 2002

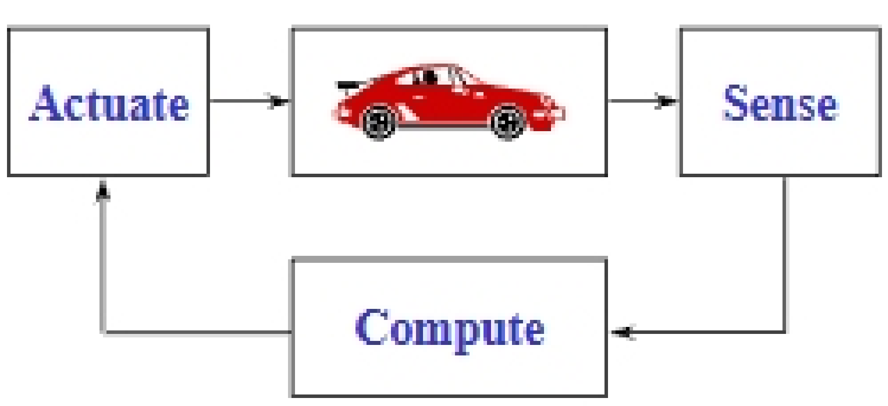
Goals:

- Describe what a model is and what types of questions it can be used to answer
- Introduce the concepts of state, dynamic, and inputs
- Provide examples of common modeling techniques: finite state automata, difference equations, differential equations, Markov chains
- Describe common modeling tradeoffs

Reading:

- K. J. Astrom, *Control Systems Design*, Sections 3.1-3.2, 3.6
- *Optional*: Astrom, Section 3.3

Review from last week



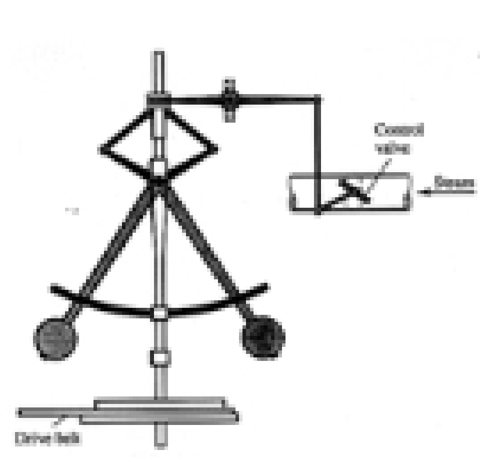
Control =

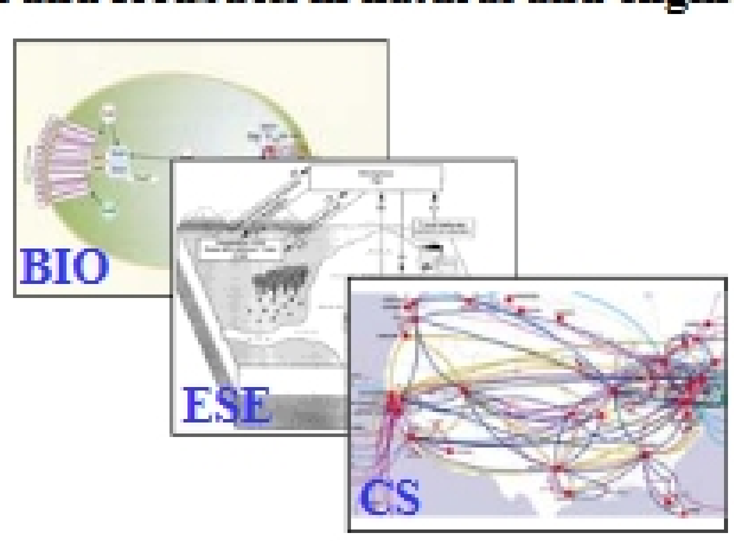
Sensing + Computation + Actuation

Feedback Principles

- Robustness to Uncertainty
- Design of Dynamics

Many examples of control and feedback in natural and engineered systems:





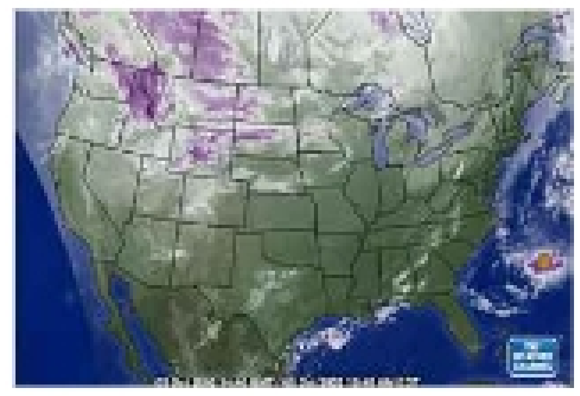
Models

Models are a mathematical *representations* of system dynamics
- Models allow the dynamics to be simulated and analyzed, without having to build the system
- Models are *never* exact, but they can be *predictive*

Models can be used in ways that the system can't
- Certain types of analysis (eg, parametric variations) can't easily be done on the actual system
- In many cases, models can be run much more quickly than the original models

The model you use depends on the questions you want to answer
- A single system may have many models
- Time and spatial scale must be chosen to suit the questions you want to answer
- *Always* formulate questions *before* building a model

Example: Weather Forecasting



- **Question 1: how much will it rain tomorrow?**
- **Question 2: will it rain in the next 5-10 days?**
- **Question 3: will we have a drought next summer?**

Different questions ⇒ different models

7 Oct 02 R. M. Murray, Caltech CDS 3

Physical Concept of State

A key concept in modeling is the concept of *state*

The *state* of a model of a dynamic system is a set of independent physical quantities, the specification of which (in the absence of excitation) completely determines the future evolution of the system

Example #1: car on a sloping road

- State: position and velocity of car
- Angle of incline is *not* a state (not part of the model of the car)
- Accelerator position is *not* a state (not *intrinsic* to the car)

Example #2: predator prey (rabbits vs foxes)

- State: number of rabbits and foxes
- Amount of rabbit food is *not* a state (not intrinsic to the ecosystem as we have defined it)
- Number of dead rabbits is *not* a state (not independent of number of live rabbits)

7 Oct 02 R. M. Murray, Caltech CDS 4

Dynamics

Dynamics **describes how the state evolves**

The *dynamics* of a model is an update rule for the system state that describes how the state evolves, as a function on the current state and any external inputs

Example #1: car on a sloping road

- Dynamics: Newton's law (F = ma)

$$m\ddot{x} = -b\dot{x} + u_{\text{engine}}(t) + u_{\text{hill}}(t)$$

- Engine force modeled as external input
- Hill modeled as external input

Example #2: predator prey

- Dynamics: empirically observed difference eqs

$$R[k+1] = R[k] + b_r R[k] - aR[k]F[k]$$
$$F[k+1] = F[k] - d_f F[k] + aR[k]F[k]$$

- *System* of difference equations

7 Oct 02 R. M. Murray, Caltech CDS 5

Inputs

Inputs describe the *external excitation* of the dynamics

- Inputs are *extrinsic* to the system dynamics (externally specified)
- Constant inputs are often considered to be *parameters*

Example #1: car on a sloping road

$$m\ddot{x} = -b\dot{x} + u_{\text{engine}}(t) + u_{\text{hill}}(t)$$

Input #1 Input #2

Example #2: predator prey

$$R[k+1] = R[k] + \mathbf{b_r(u)}R[k] - aR[k]F[k]$$
$$F[k+1] = F[k] - d_f F[k] + aR[k]F[k]$$

- Rabbit food can either be a parameter (if constant) or an external input (if nonconstant)

7 Oct 02 R. M. Murray, Caltech CDS 6