CDS 101: Lecture 8.1
Frequency Domain Design

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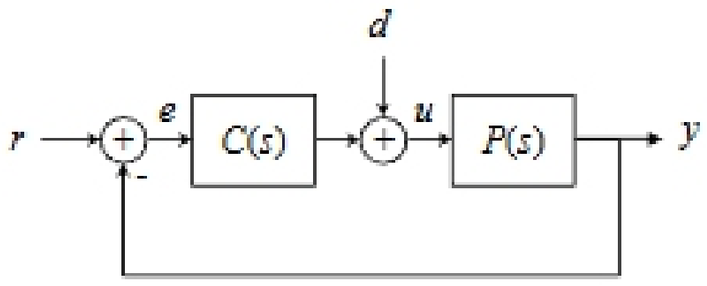
Goals:

- Describe the use of frequency domain performance specification
- Show how to use "loop shaping" to achieve a performance specification
- Work through a detailed example of a control design problem

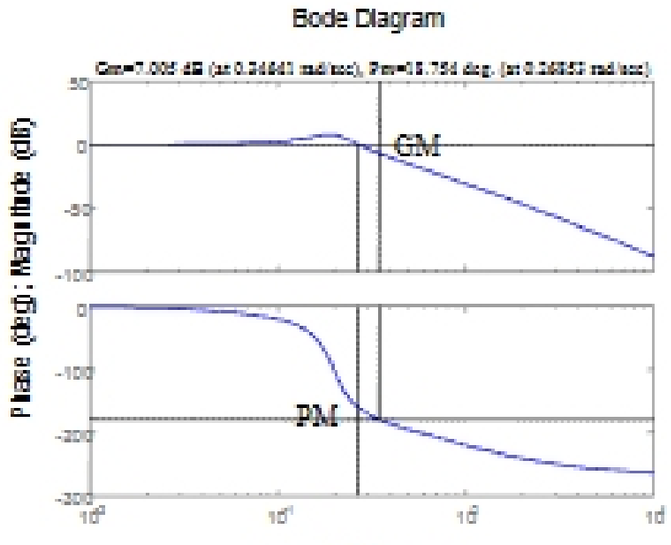
Reading:

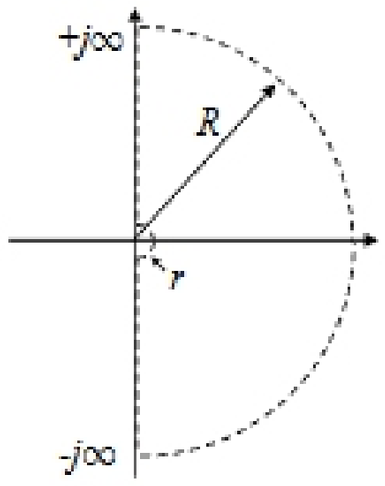
- No new reading this week
- *Advanced*: Lewis, Chapter 12

Review from Last Week



- Nyquist criteria for loop stability
- Gain, phase margin for robustness





Thm (Nyquist).

P # RHP poles of $L(s)$

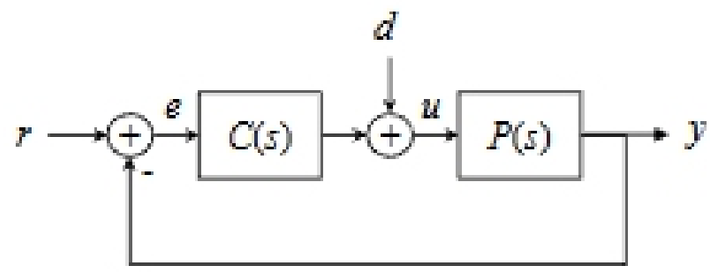
N # CW encirclements

Z # RHP zeros

$$Z = N + P$$

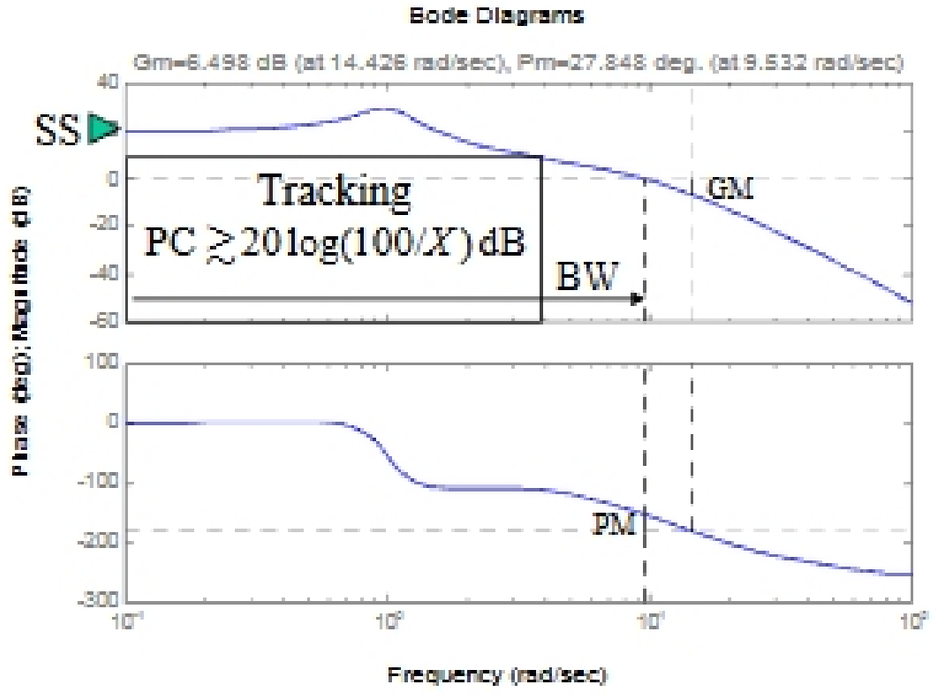
Frequency Domain Performance Specifications

Specify bounds on the loop transfer function to guarantee desired performance



$$L(s) = P(s)C(s)$$

$$H_{er} = \frac{1}{1+L} \qquad H_{yr} = \frac{L}{1+L}$$



- Steady state error:

  $H_{er}(0) = 1/(1+L(0)) \approx 1/L(0)$

  $\Rightarrow$ zero frequency ("DC") gain ▶
- Bandwidth: assuming ~90° phase margin

  $$\frac{L}{1+L}(j\omega_c) \approx \left|\frac{1}{1+j}\right| = \frac{1}{\sqrt{2}}$$

  $\Rightarrow$ sets crossover freq →
- Tracking: X% error up to frequency $\omega_t \Rightarrow$ determines gain bound $(1 + PC > 100/X)$ □

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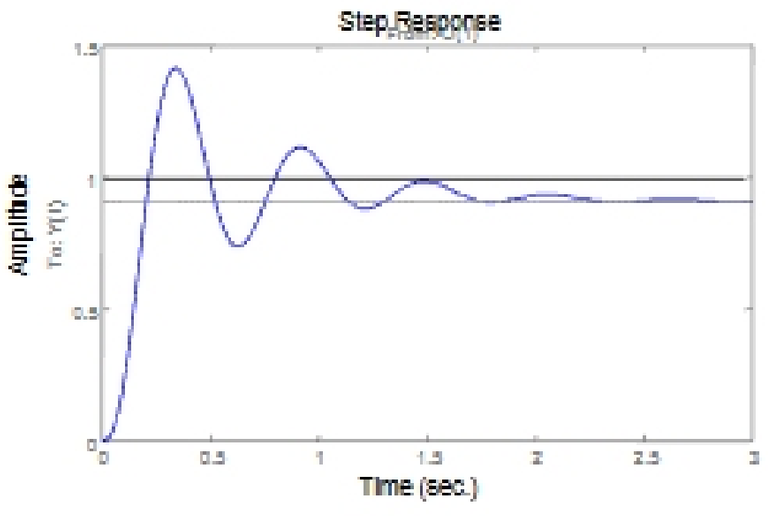
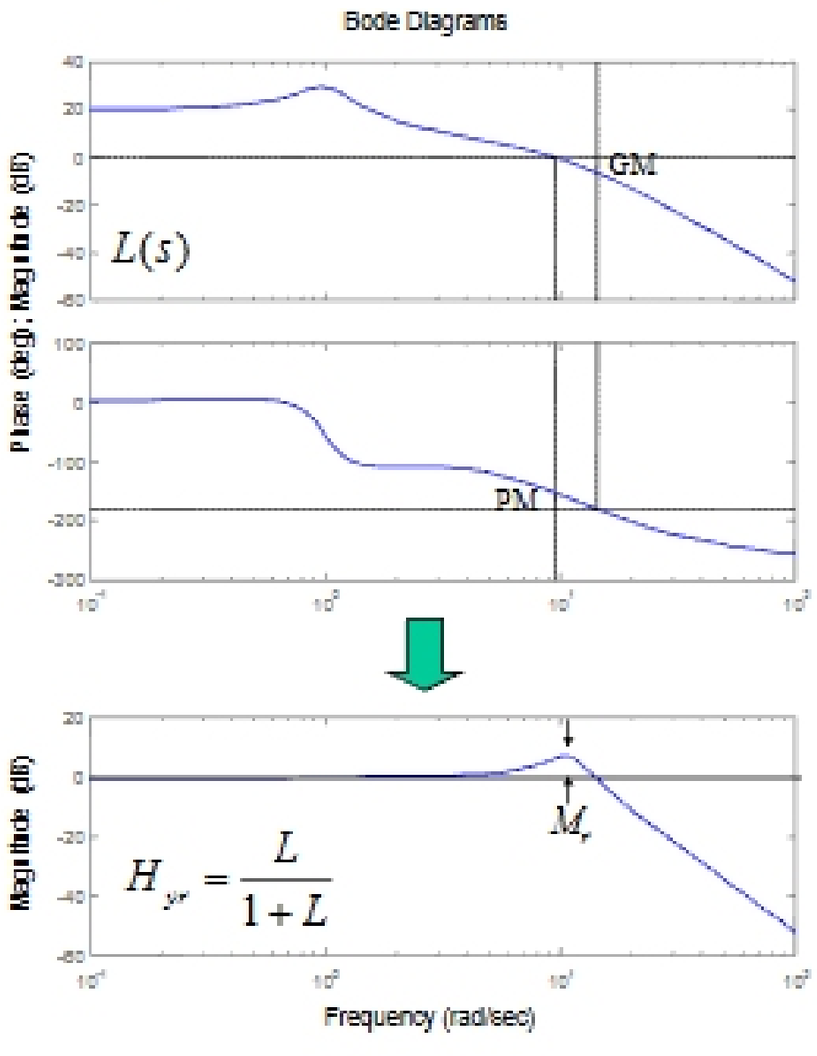
Relative Stability

Relative stability: how stable is system to disturbances at certain frequencies?

- System can be stable but still have very bad response at certain frequencies
- Typically occurs if system has low phase margin $\Rightarrow$ get resonant peak in closed loop (M_r) + poor step response
- Solution: specify minimum phase margin. Typically 45° or more





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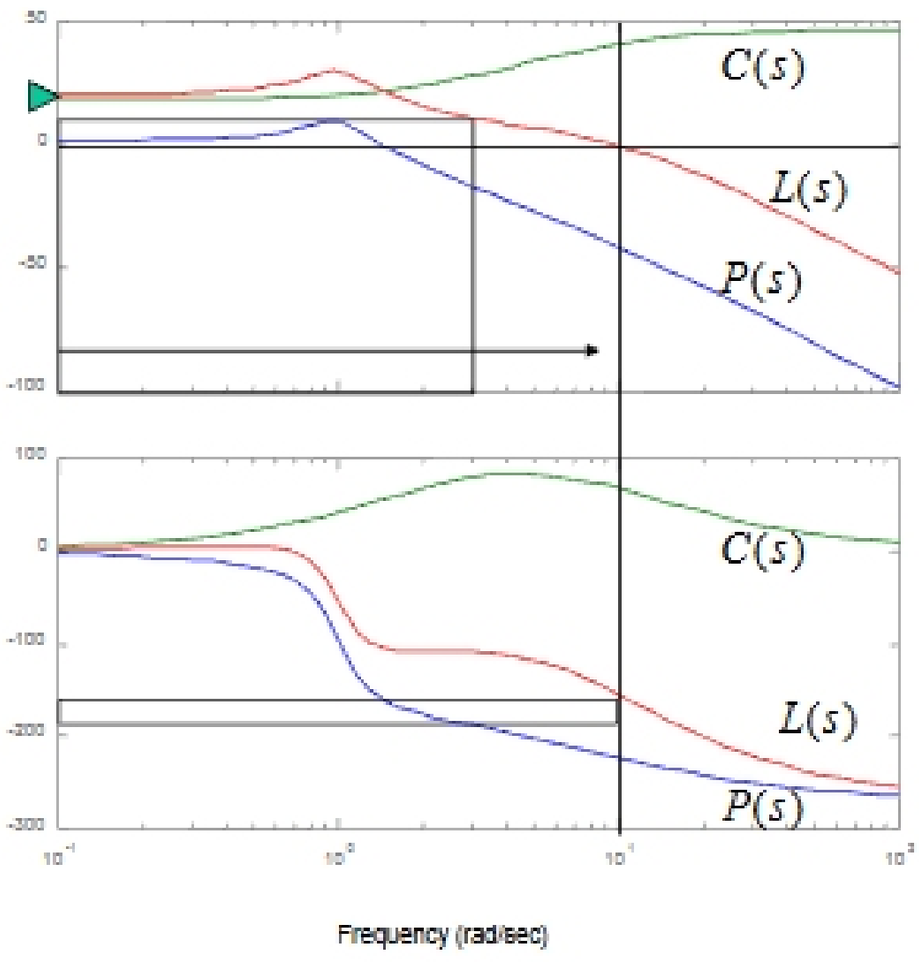
Overview of Loop Shaping



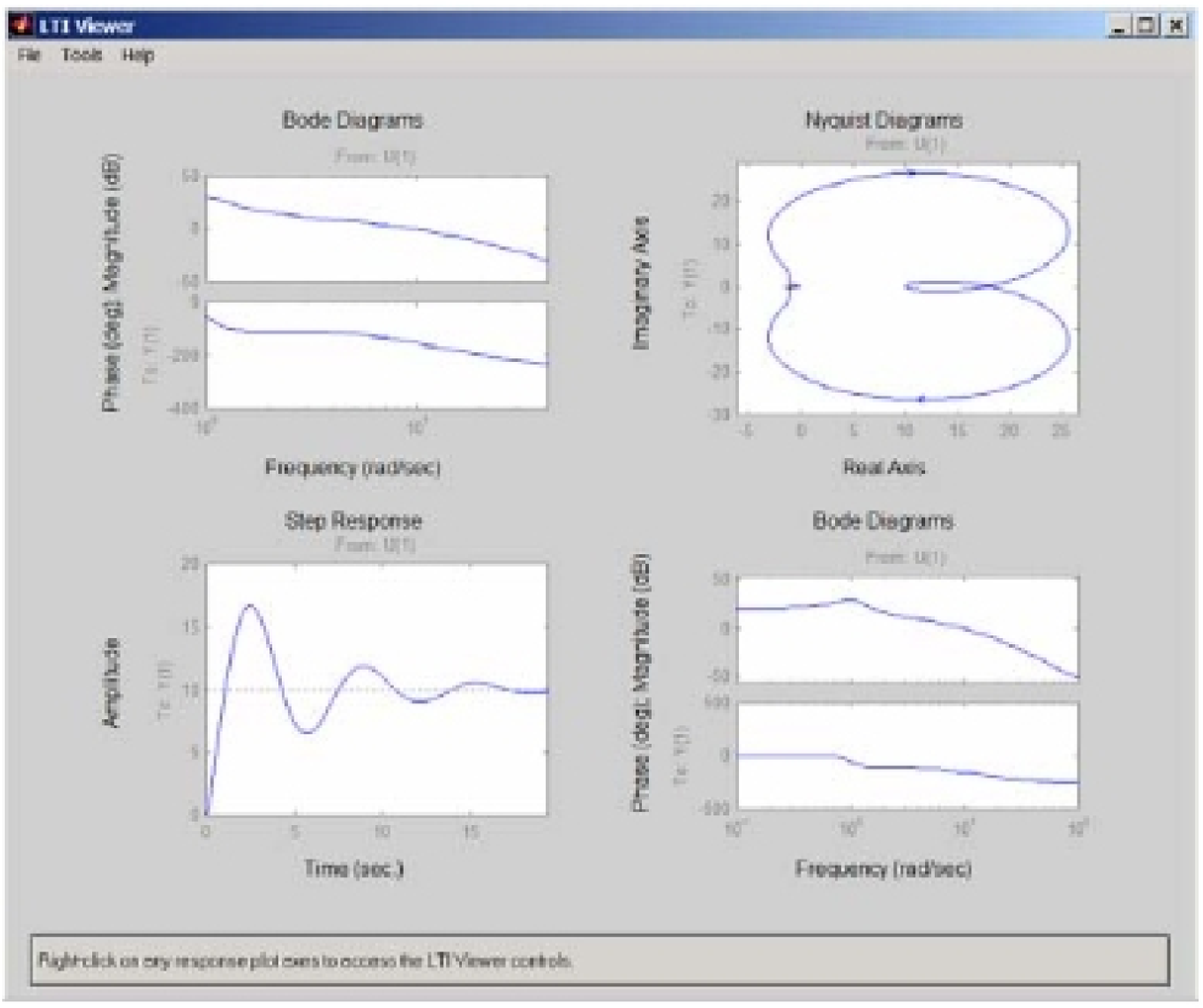
Performance specification

- Steady state error
- Tracking error
- Bandwidth
- Relative stability

Approach: "shape" loop transfer function using $C(s)$

- $P(s)$ + specifications given
- $L(s) = P(s)\,C(s)$
  - Use $C(s)$ to choose desired shape for L(s)
- Important: can't set gain and phase independently

LTIView



MATLAB LTIView

- Allows simultaneous view of up to six plots
- Can apply to any system in the MATLAB workspace
- Useful for seeing how the various concepts relate to each others

Caution

- Doesn't allow much control over details of each plot