CDS 101: Lecture 7.1
Loop Analysis of Feedback Systems

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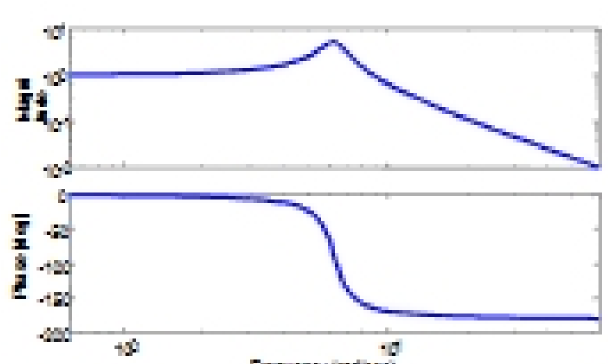
Goals:
- Show how to compute closed loop stability from open loop properties
- Describe the Nyquist stability criterion for stability of feedback systems
- Define gain and phase margin and determine it from Nyquist and Bode plots

Reading:
- Åström and Murray, *Analysis and Design of Feedback Systems*, Ch 7
- *Advanced*: Lewis, Chapter 7

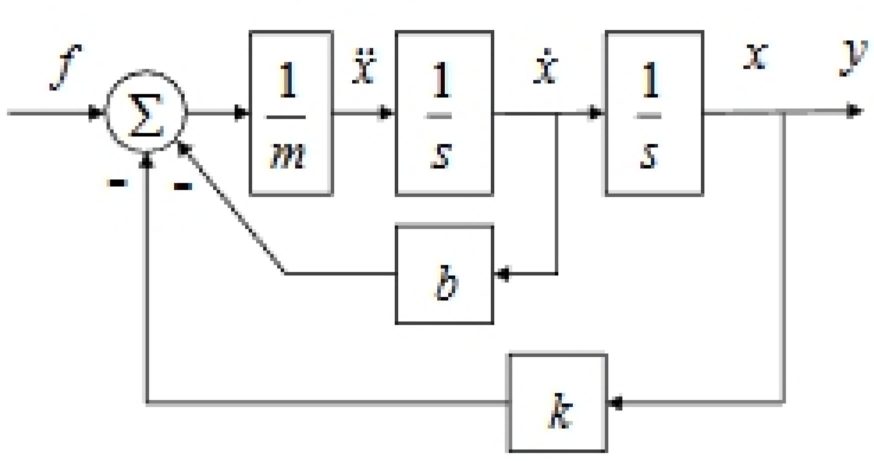
Review from Last Week

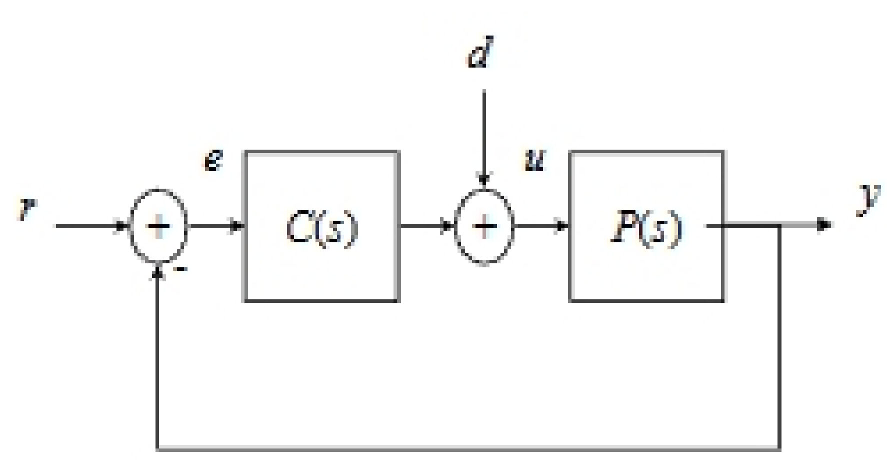
$u = A\sin(\omega t)$ → $\dot{x} = Ax + Bu$, $y = Cx + Du$, $x(0) = 0$ → $y_{ss} = |H(j\omega)| A \cdot \sin(\omega t + \angle H(j\omega))$



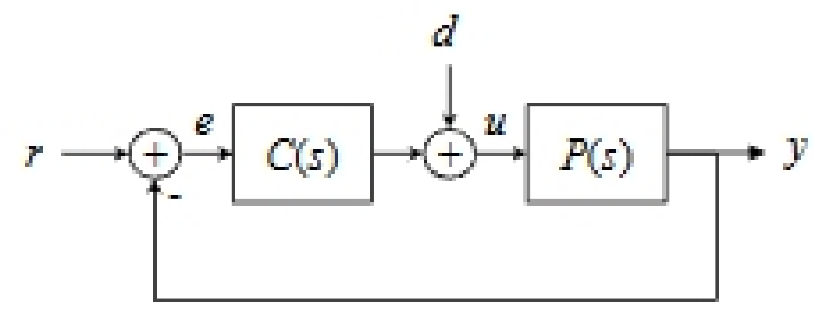
$$H(s) = C(sI - A)^{-1}B + D$$

$$H_{y_2u_1} = H_{y_2u_2} H_{y_1u_1} = \frac{n_1 n_2}{d_1 d_2}$$





Closed Loop Stability



Q: how do open loop dynamics affect the closed loop stability?

- Given open loop transfer function $C(s)P(s)$ determine when system is stable

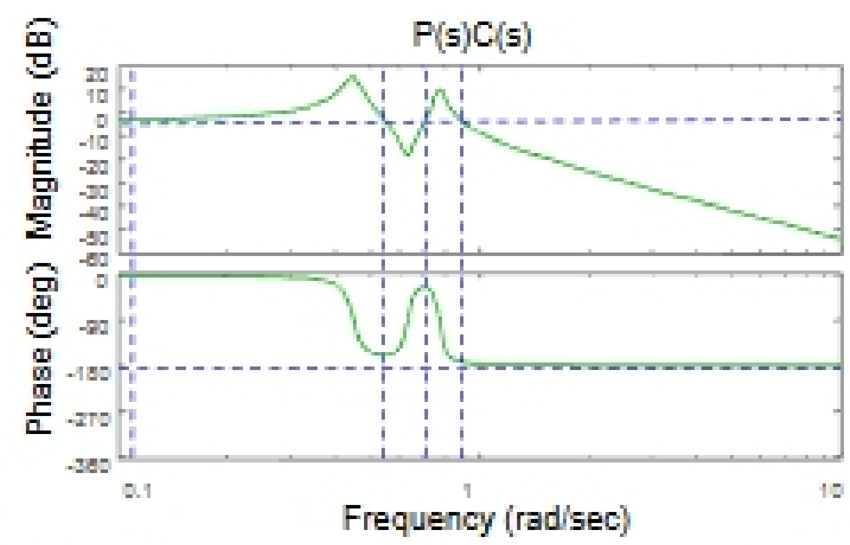
Brute force answer: compute poles closed loop transfer function

$$H_{yr} = \frac{PC}{1+PC} = \frac{n_p n_c}{d_p d_c + n_p n_c}$$

- Poles of H_{yr} = zeros of $1 + PC$
- Easy to compute, but not so good for design

Alternative: look for conditions on *PC* that lead to instability

- Example: if $PC(s) = -1$ for some $s = j\omega$, then system is *not* asymptotically stable
- Condition on *PC* is much nicer because we can *design* $PC(s)$ by choice of $C(s)$
- However, checking $PC(s) = -1$ is not enough; need more sophisticated check

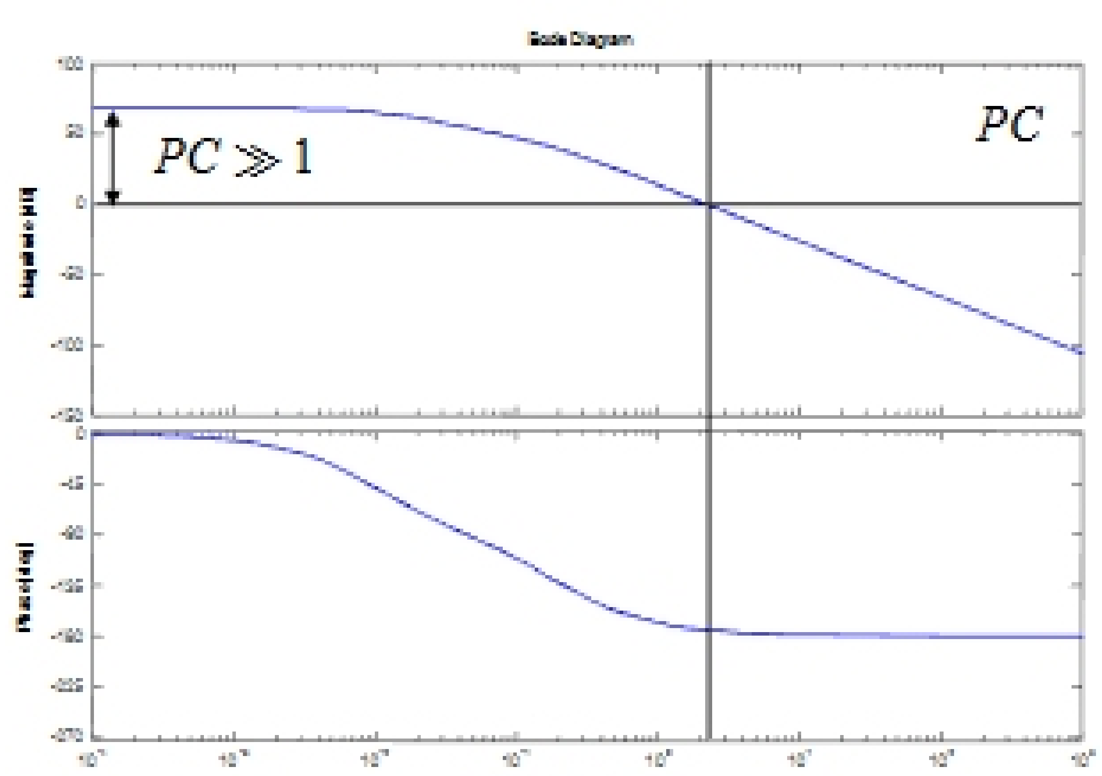


Game Plan: Frequency Domain Design

Goal: figure out how to *design* $C(s)$ so that $1+C(s)P(s)$ is stable *and* we get good performance

$$H_{yr} = \frac{PC}{1+PC}$$

- Poles of H_{yr} = zeros of $1 + PC$
- Would also like to "shape" H_{yr} to specify performance at differenct frequencies

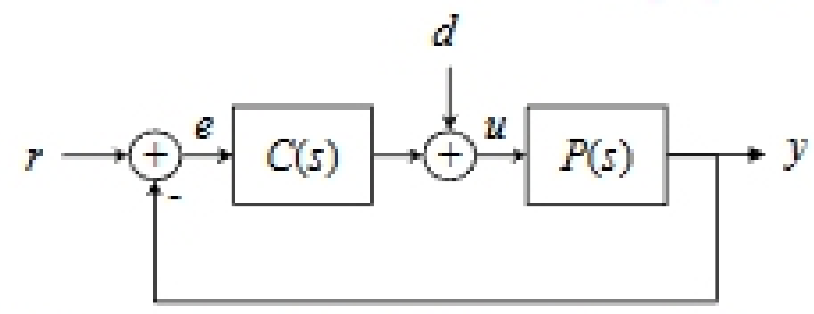


- Low frequency range:

$$PC \gg 1 \;\Rightarrow\; \frac{PC}{1+PC} \approx 1$$

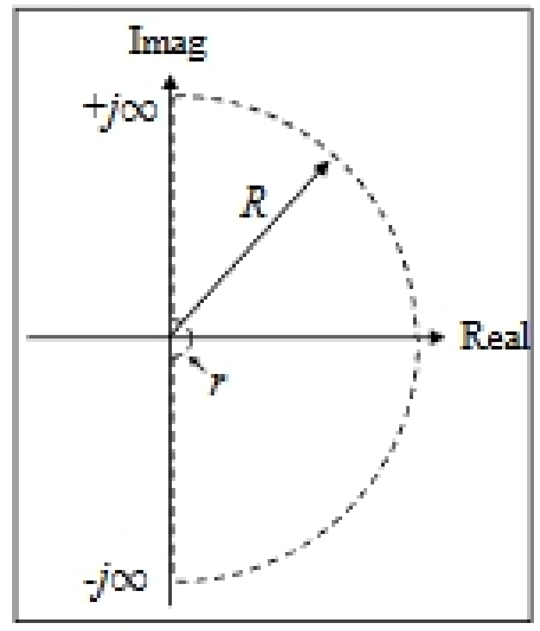
  (good tracking)
- Bandwidth: frequency at which closed loop gain = ½ ⇒ open loop gain ≈ 1
- Idea: use $C(s)$ to *shape* *PC* (under certain constraints)
- Need tools to analyze stability and performance for closed loop given *PC*

Nyquist Criterion



Determine stability from (open) loop transfer function, $L(s) = P(s)C(s)$.

- Use "principle of the argument" from complex variable theory (see reading)



- Nyquist "D" contour
- Take limit as $r \to 0$, $R \to \infty$
- Trace from $-\infty$ to $+\infty$ along imaginary axis

Thm (Nyquist). Consider the Nyquist plot for loop transfer function $L(s)$. Let

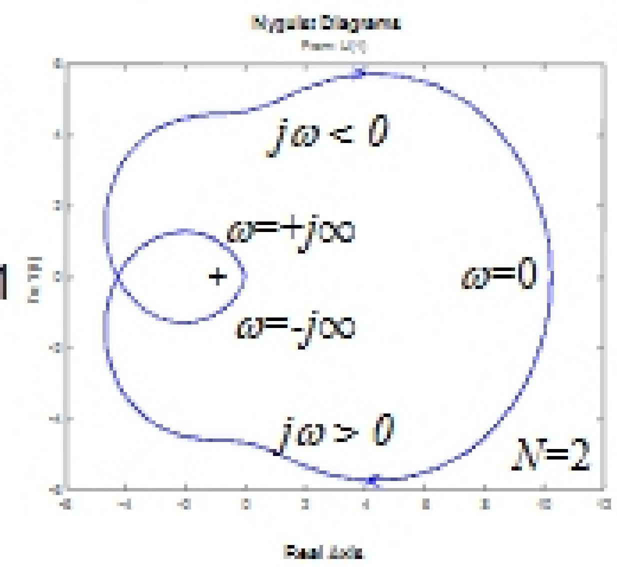
P # RHP poles of $L(s)$

N # clockwise encirclements of -1

Z # RHP zeros of $1 + L(s)$

Then

$$Z = N + P$$



- Trace frequency response for $L(s)$ along the Nyquist "D" contour
- Count net # of clockwise encirclements of the -1 point

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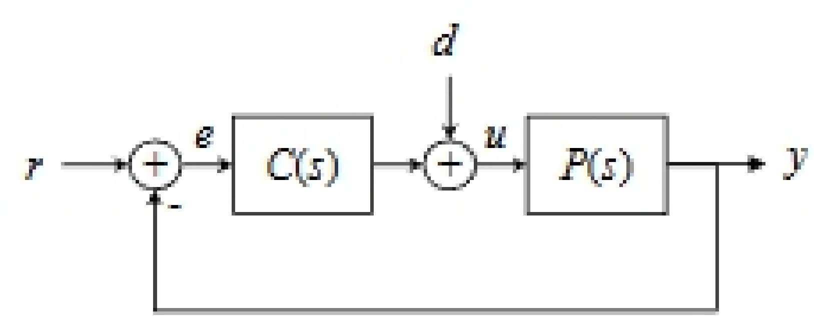
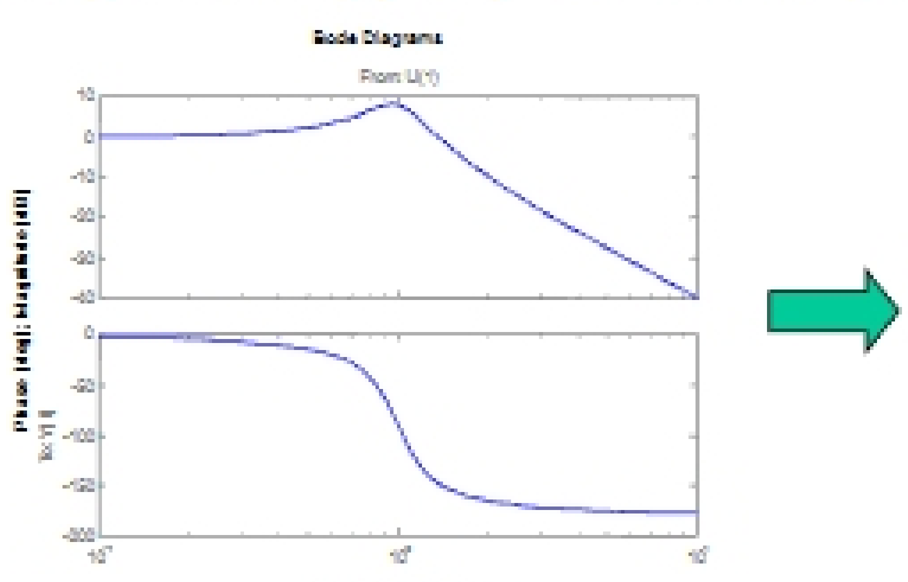
Simple Interpretation of Nyquist



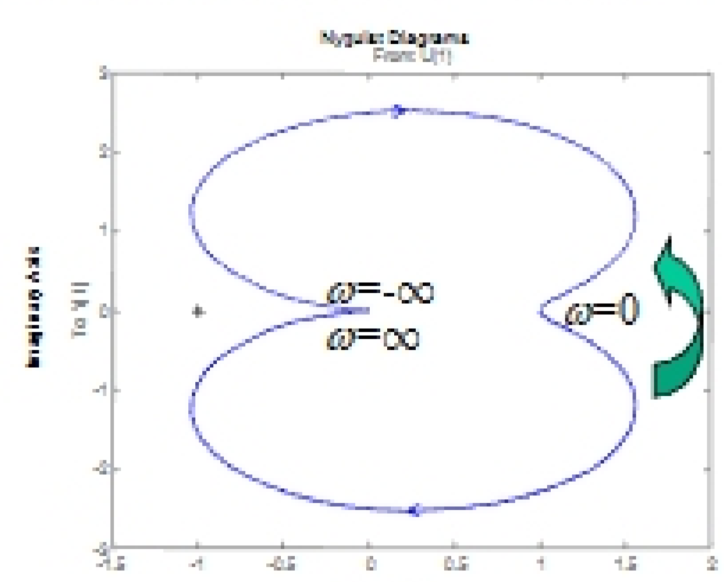
Basic idea: avoid positive feedback

- If $L(s)$ has 180° phase (or greater) and gain greater than 1, then signals are amplified around loop
- Use when phase is monotonic
- General case requires Nyquist

Can generate Nyquist plot from Bode plot + reflection around real axis



bode(sys)



nyquist(sys)

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