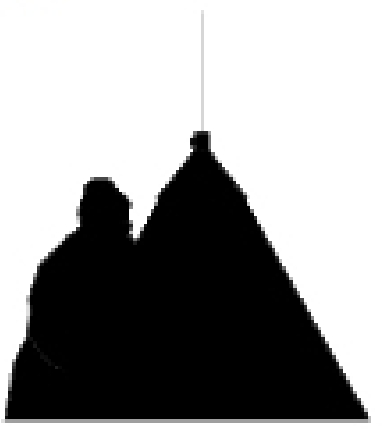
Great Theoretical Ideas In Computer Science

Steven Rudich | CS 15-251 Spring 2005

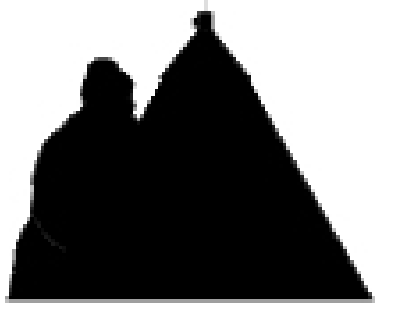
Lecture 15 March 1, 2005 | Carnegie Mellon University

Problem Solving: Where does the "aha!" come from?

Relax .....



Relax, I am just going to ask you a Microsoft interview question.



Do You Understand The Question?

Four guys want to cross a bridge that can only hold two people at one time. It is pitch dark and they only have one flashlight, so people must cross either alone or in pairs (bringing the flashlight). Their walking speeds allow them to cross in 1, 2, 5, and 10 minutes, respectively. Is it possible for them to all cross in 17 minutes?

You have one minute to solve this problem

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So what is the answer?

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Intuitive, But False

"10 + 5 + 2 + 1 = 18, so the four guys just can't cross in 17 minutes"

"Even if the fastest guy is the one to shuttle the others back and forth – you use at least 10 + 5 + 2 + 1 > 17 minutes"

Keep track of what you actually know – remember what you merely suspect.

"10 + 5 + 2 + 1 = 18, so it would be weird if the four guys could cross in 17 minutes"

"~~even~~ If we use the fastest guy to shuttle the others, they take too long."

Tagging Strategy

As you talk to yourself, make sure to tag assertions with phrases that denote degrees of conviction

Keep track of what you actually know – remember what you merely suspect.

"10 + 5 + 2 + 1 = 18, so it would be weird if the four guys could cross in 17 minutes"

"~~even~~ If we use the fastest guy to shuttle the others, they take too long."

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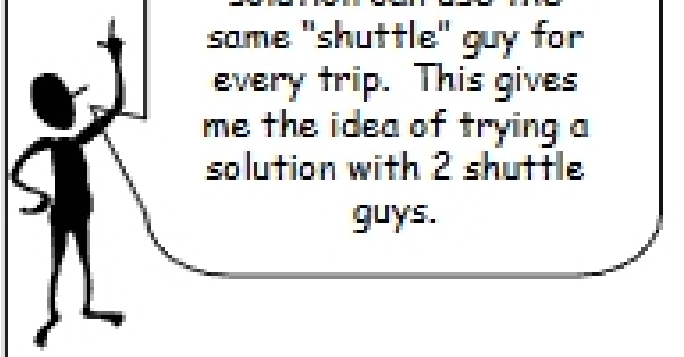
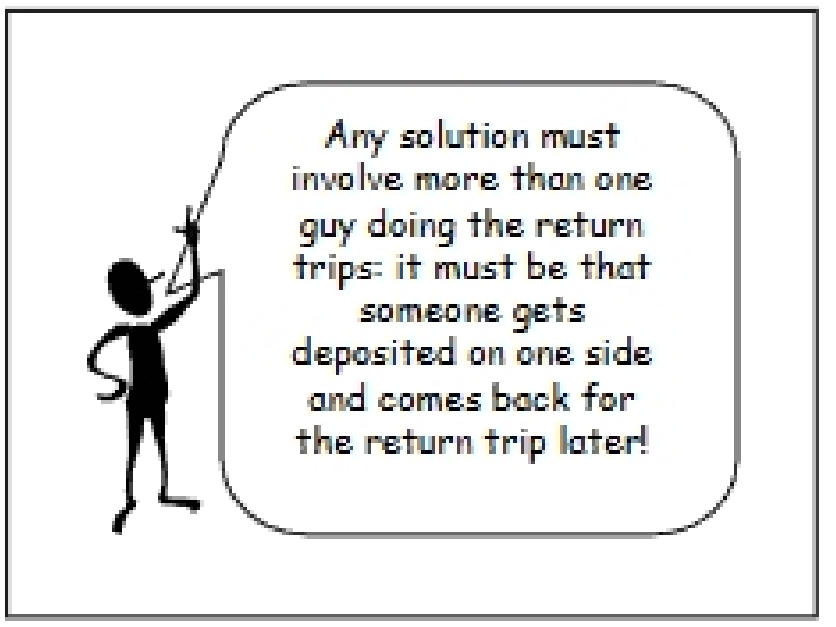
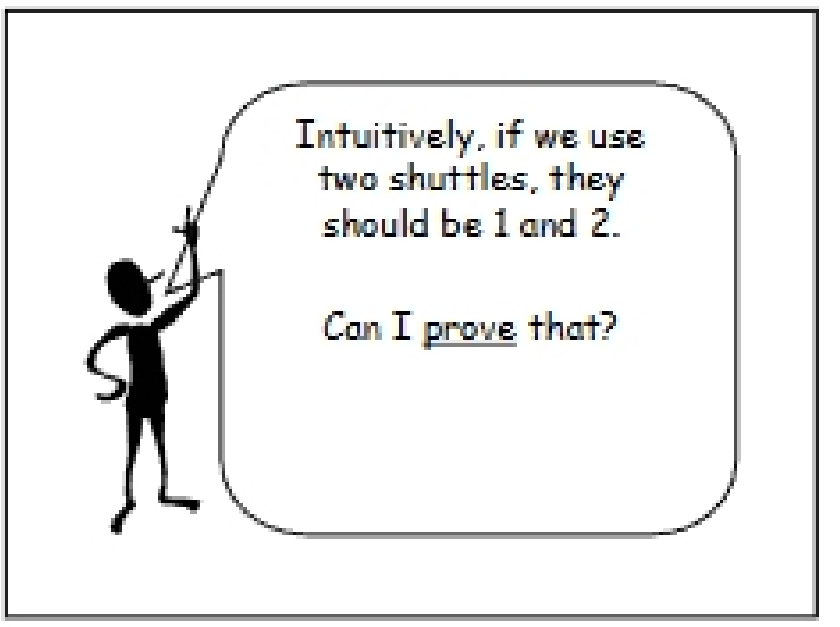
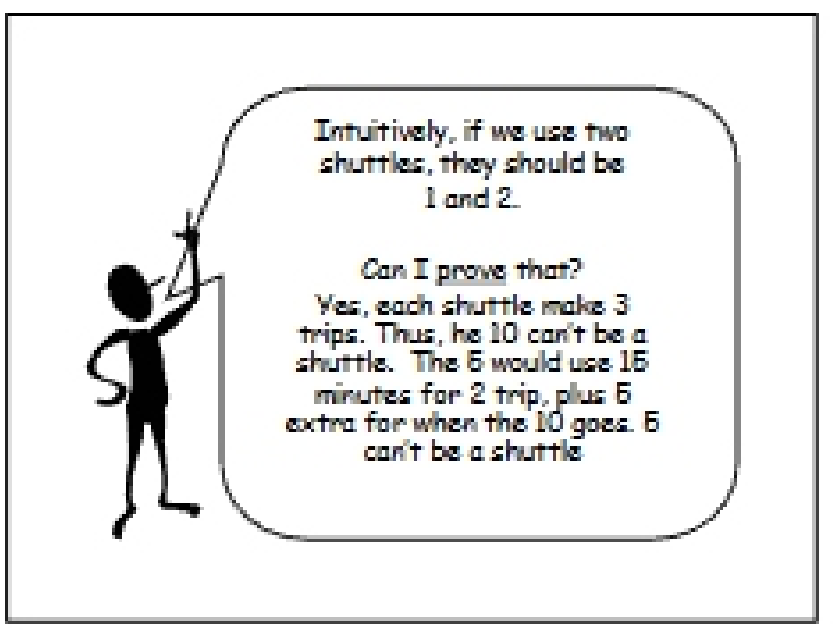
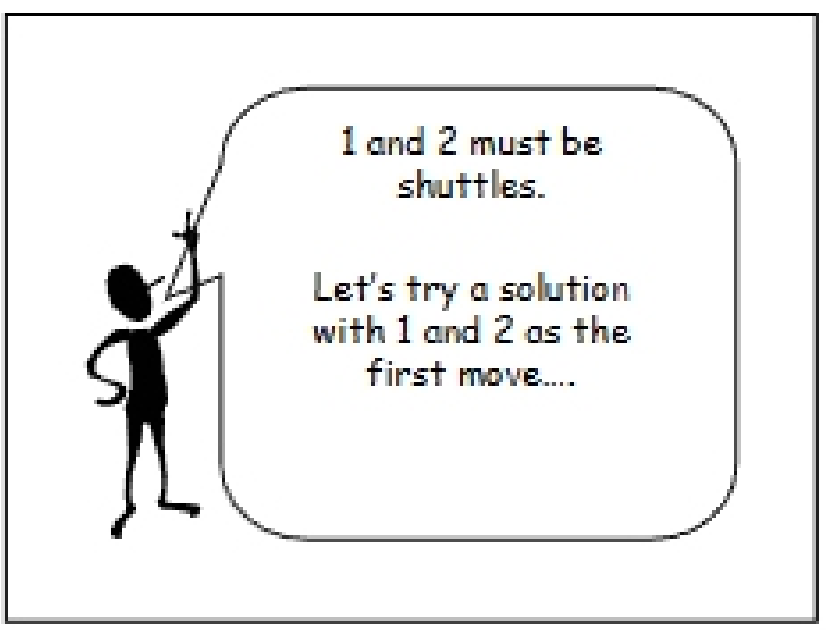
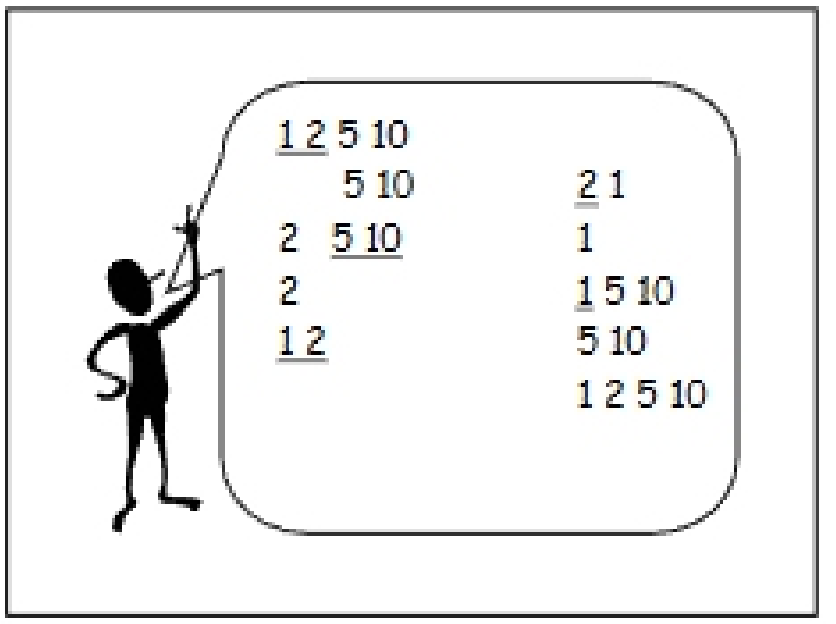


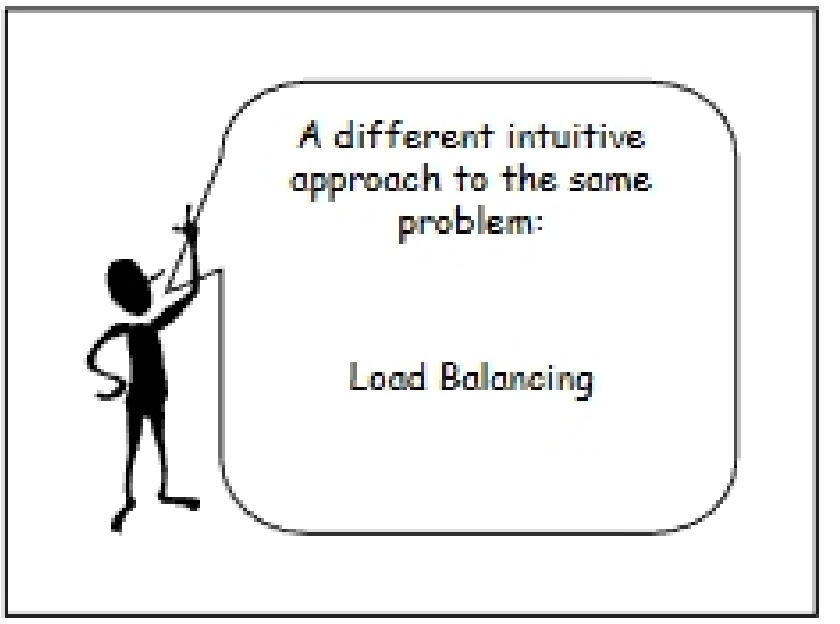

Any solution must involve more than one guy doing the return trips: it must be that someone gets deposited on one side and comes back for the return trip later!


Intuitively, if we use two shuttles, they should be 1 and 2.
Can I prove that?


Intuitively, if we use two shuttles, they should be 1 and 2.
Can I prove that?
Yes, each shuttle make 3 trips. Thus, he 10 can't be a shuttle. The 5 would use 15 minutes for 2 trip, plus 5 extra for when the 10 goes. 5 can't be a shuttle


1 and 2 must be shuttles.
Let's try a solution with 1 and 2 as the first move....


1 2 5 10
5 10
2 5 10
2
1 2
2 1
1
1 5 10
5 10
1 2 5 10


A different intuitive approach to the same problem:
Load Balancing