Lecture 09: Filter Networks

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Abstract

Discussion of Low-pass filters, High-pass filters, Band-pass filters, Band-rejection filters.

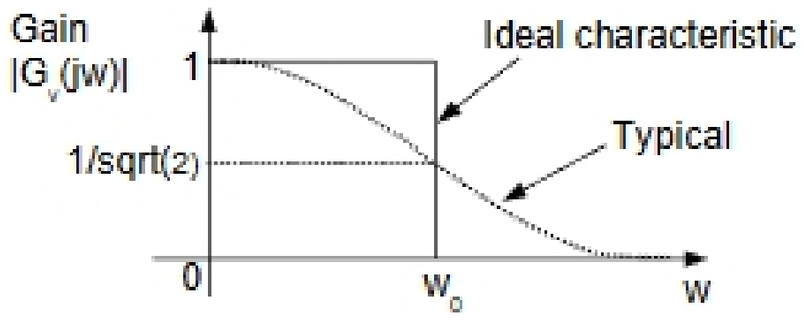

Figure 1: Low-pass filter.

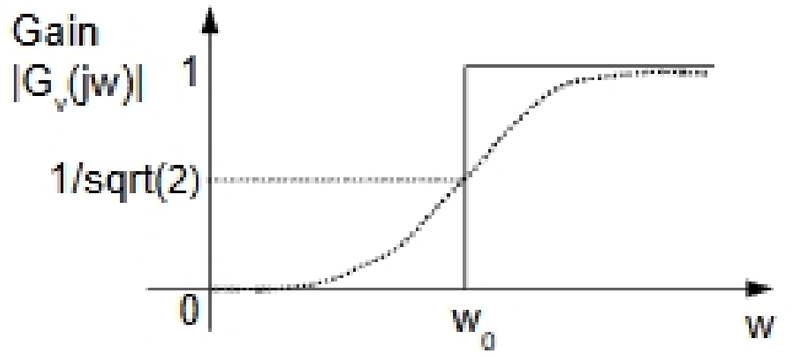

Figure 2: High-pass filter.

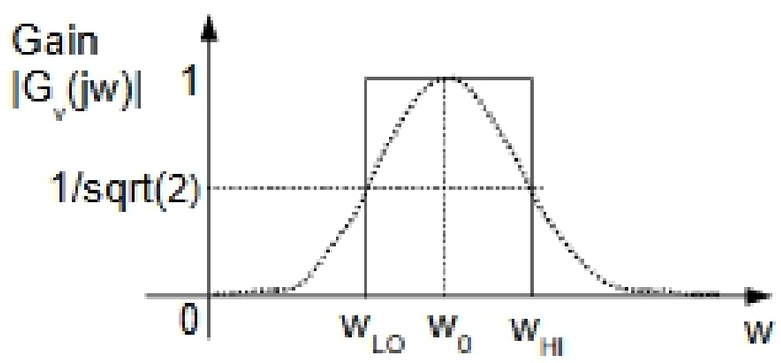

Figure 3: Band-pass filter.

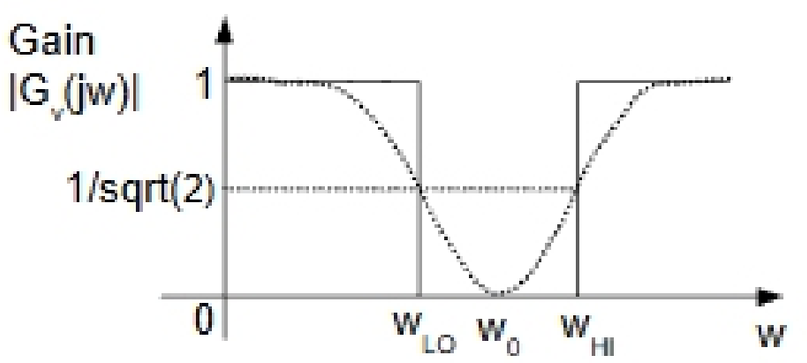

Figure 4: Band-rejection filter.

1 Low-pass filter network

A simple low-pass filter:

$$G_v(j\omega) = \text{gain} = \frac{V_O}{V_1} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}$$

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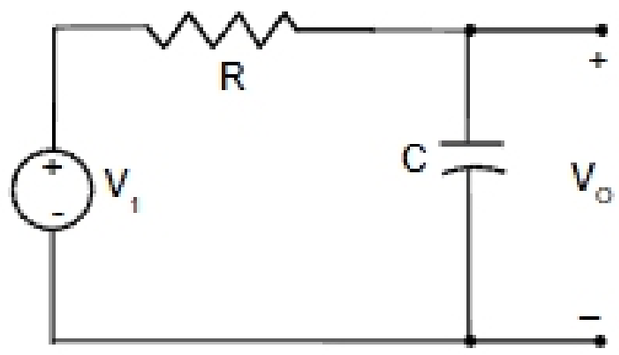

Figure 5: Low-pass filter.

$$G_V(j\omega) = \frac{1}{1 + j\omega RC} \tag{1}$$

$$= \frac{1}{1 + j\omega\tau} \tag{2}$$

where,

$$\tau = RC$$

$$\text{Amplitude:} \quad M(\omega) = \frac{1}{\sqrt{1 + (\omega\tau)^2}} \tag{3}$$

$$\text{Phase:} \quad \Phi(\omega) = -\arctan(\omega\tau) \tag{4}$$

At **break frequency** (also called **half-power frequency**), $\omega_b\tau = 1$, $\omega_b = \frac{1}{\tau}$

$$(M)_{\omega=\omega_b} = \frac{1}{\sqrt{2}} \tag{5}$$

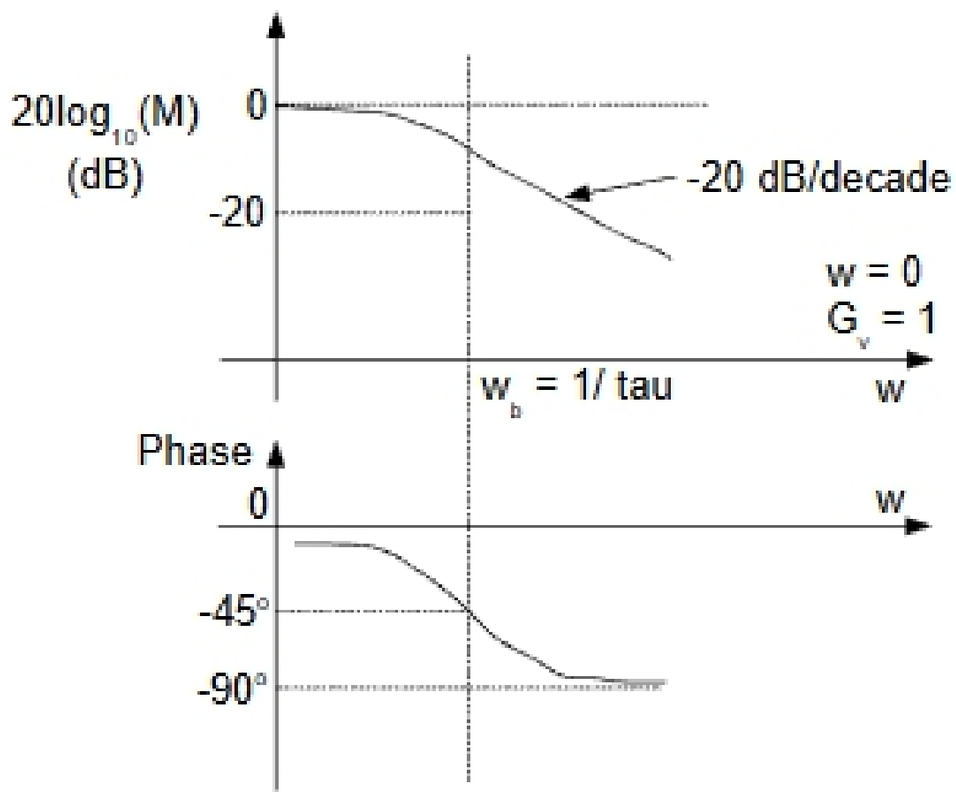

Figure 6: Low-pass filter characteristics.

2 High-pass filter network

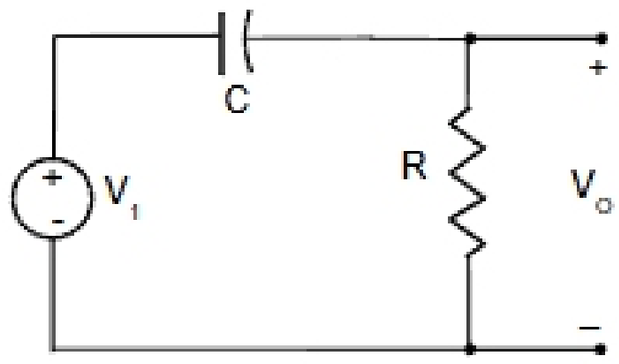

Figure 7: High-pass filter.

$$
\begin{aligned}
G_v(j\omega) &= \frac{R}{R+\frac{1}{j\omega C}} \\
&= \frac{j\omega RC}{1+j\omega RC} \\
&= \frac{j\omega\tau}{1+j\omega\tau}; \quad \tau = RC \\
M(\omega) &= \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}
\end{aligned}
$$

$$\Phi(\omega) = \frac{\pi}{2} - \arctan(\omega\tau)$$

$$\text{Half-power frequency} = \omega_b = \frac{1}{\tau}$$

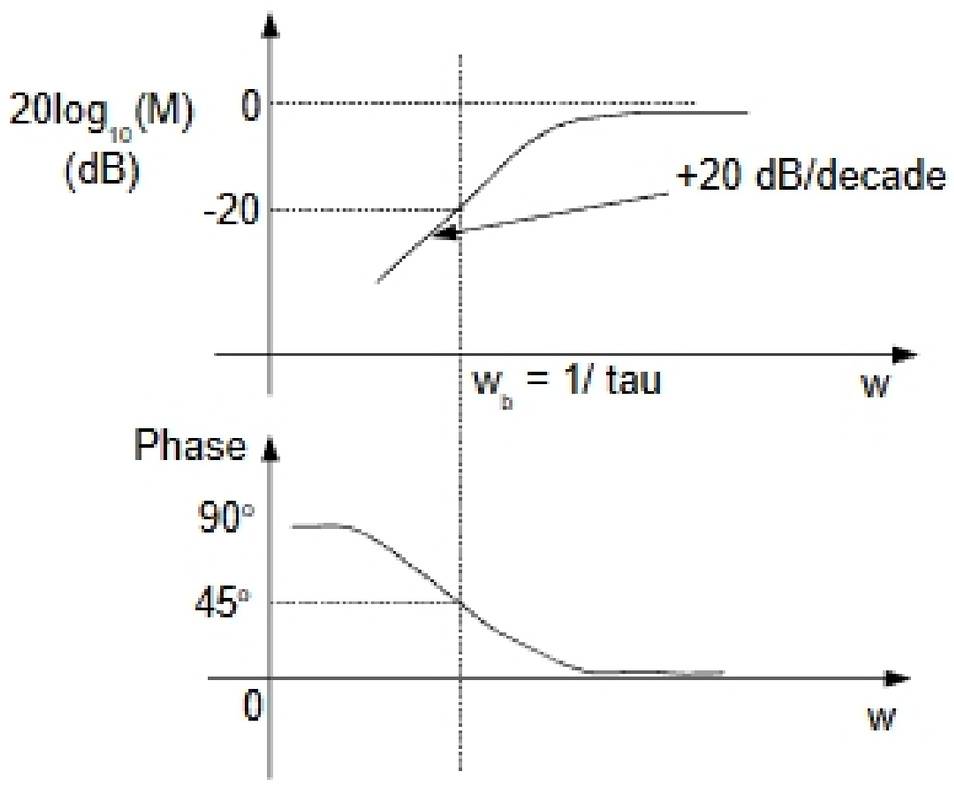

Figure 8: High-pass filter characteristics.