

Final Exam PH314

08 December 2006

Good luck! Don't sweat!

1. In Figure 25.37 using the rotation curve of M31 (the Andromeda Galaxy), calculate the mass within a 1" diameter of the center of the galaxy. Write your answer in solar masses. Use the formula $F(\text{centripetal}) = F(\text{gravity})$ and assume all the mass is located in the center. Note that the Figure has real data, so that you must roughly estimate the average rotation amplitude.
2. The Virial Theorem states that $E(\text{kinetic energy}) = -1/2 U(\text{gravitational potential energy})$. Show that this implies that for a relaxed cloud of stars, the virial mass is $M(\text{virial}) = 6 \cdot R \cdot \sigma^2 / G$ where R is the radius of the cloud. Remember that the measured velocity dispersion σ is related to the total velocity dispersion as $\sigma^2 = v_x^2 + v_y^2 + v_z^2$. (Remember problem 14.5 which you did in the homework!). Calculate the virial mass of the black hole in M31 assuming that all the mass within 0.1" of the center is due to the black hole using Figure 25.37. Give the answer in solar masses.
3. Show that if every piece of space expands at a constant rate, you must have a Hubble law $v(t) = H_0 \cdot r(t)$ where $v(t)$ is the velocity between two galaxies, $r(t)$ is the distance between the galaxies at a given time, and H_0 is a constant of proportionality. Can $v(t) > c$, the speed of light?
4. The local value of the Hubble constant is $71 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Find the distance in mega-parsecs to the Coma cluster of galaxies at a redshift of $z=0.024$.
5. The Virgo Cluster of galaxies is located at 20 Mpc. The observed velocity of the cluster is $v_{\text{obs}} = 1170 \text{ km s}^{-1}$. Compare this to the expected velocity from the Hubble law, v_{Hubble} . Give an explanation why these numbers are not the same and why $v_{\text{obs}} < v_{\text{Hubble}}$. (Hint: Consider the sign of the velocities for the expansion of the Universe and the gravitational attraction of nearby galaxies).
6. Assuming that the Universe has a constant expansion over all time, use the Hubble constant to estimate the age of the Universe. Give the answer in billions of years.
7. Derive the equation $\Omega(t) = 1 + kc^2/(dR/dt)^2$ for a one-component universe of pressureless dust. Use the Friedmann equation 29.10 and the definition of $\Omega(t)$ given in 29.18. Using the Friedmann equation again (29.11), show that as $R \rightarrow 0$, $dR/dt \rightarrow \infty$. What happens to $\Omega(t)$ as $dR/dt \rightarrow \infty$? What does this say about the nature of the early universe, that is, the geometry and velocity? Note: since $dR/dt > 0$ today, you can assume that $dR/dt > 0$ as $R \rightarrow 0$.

8. Consider equation 29.53 where $R^{3(1+w)} \cdot \rho = \rho_0$. Show that if $w < -1$, as the Universe expands with $R(t)$ increasing, the density $\rho(t)$ must also increase. Now consider equation 29.51. Using the equation of state (29.52), show that, given $w < -1$, the acceleration must be positive and increasing as the Universe expands. (Hint: consider the product $R(t) \cdot \rho(t)$ in equation 29.51). Explain in simple words what the fate of the Universe will be under these circumstances.
9. The pressure of a photon gas is $P = 1/3 \cdot u_{\text{rad}}$ where $u_{\text{rad}} = a \cdot T^4$, the energy density of a photon gas. Since $E = mc^2$, one can think of the photon energy density as (mc^2/volume) , or ρc^2 . Thus, according to equation 29.52, $w = 1/3$ for a photon gas. Use equation 29.53, $R^{3(1+w)} \cdot \rho = \rho_0$, to show that $R(t) \cdot T(t) = T_0$. What was the temperature of the Universe at recombination when $z = 1089$? Hint: don't forget eqn 29.4.
10. What is the present phase of the moon?