Your name: ______________________________

STATISTICS 204 FALL 2007 FINAL

This is a 3-hour in-class exam. There are 7 problems: do as many problems as you can. You are not expected to do them all. Most questions do not require long calculations. Students are allowed a calculator and notes *in their own handwriting* but not other written material.

Write on "question" side of paper only - will be scanned! Extra sheets available. Write full name above, and initials at the ________ on top right of each page and any extra sheets.

1. Let Y have Poisson(μ) distribution. Conditional on $Y = y$ let X have Poisson(y) distribution. Find the probability generating function of $X + Y$.

2. Consider a Poisson point process on the plane with intensity function $\lambda(x_1, x_2) = x_1^2 + x_2^2$. Suppose the points represent trees in a forest.
(a) Let D be the distance from the origin to the closest tree. Find the probability density function of D.
(b) Suppose that a fire burns down each tree with probability 1/3, independently for different trees. Let D^* be the distance from the origin to the closest tree that is not burned down. Find the probability density function of D^*.

3. Consider a Galton-Watson branching process whose offspring distribution has mean 1 and variance $0 < \sigma^2 < \infty$. Let $(X_n^{(K)},\ n = 0, 1, 2, \ldots)$ be the size of the n'th generation when the intial size is $X_0^{(K)} = K$. Rescale to define

$$Y_{n\delta_K}^{(K)} = K^{-1} X_n^{(K)}$$

that is, regard each generation as living for δ_K of a standard time unit. With appropriate choice of δ_K, for large K the process $Y^{(K)}$ approximates a certain diffusion Y. Find an appropriate definition of δ_K and find the drift rate $\mu(y)$ and variance rate $\sigma^2(y)$ for the diffusion Y.