ECON 201B - Game Theory

Suggested Answers - Final Exam

March 24, 2006

1 Hunter-Gatherer

Two players must decide whether to be hunters or gathers. If both are hunters, both receive 0; if both are gatherers both receive 1. If one is a hunter and one a gatherer, the hunter receives 3 and the gatherer 2.

a) *Find the normal form of this game.*

	H	G
H	0, 0	**3, 2**
G	**2, 3**	1, 1,

b) *Find the Nash equilibrium of this game.*

Best responses in pure strategies are denoted in bold in the matrix above. There are two NE in pure strategies (H, G) and (G, H). There is also a mixed NE in which both players randomizes 50-50 (i.e. $(\frac{1}{2}G + \frac{1}{2}H, \frac{1}{2}G + \frac{1}{2}H)$). **NE Payoffs to player 1 in each case are respectively 3, 2 and 1.5.**

c) *Are there any dominated strategies?*

There is **NO** strategy strictly dominated (i.e. no strategy strictly preferred by a player regardless of what the other player plays).

d) *Find the pure and mixed Stackelberg equ. in which player 1 moves first.*

The highest NE payoff is also the highest possible payoff attainable by player 1 (i.e. 3). Hence both the **pure and mixed Stackelberg equilibrium deliver 3** as well (when player 1 commits to play H and player 2 reacts by playing G).

e) *Find the minmax for both players.*

The definition of minmax (for player 1) is $\mathbf{m}^1 = \min_{a^2}\left[\max_{a^1} u^1(a^1, a^2)\right]$. In this game

a^2	$a^1 \in BR^1(a^2)$	Payoffs
$Pr(H) < \frac{1}{2}$	G	$(1.5, 2]$
$Pr(H) = \frac{1}{2}$	G, H, mix	1.5
$Pr(H) > \frac{1}{2}$	H	$(1.5, 3]$

being $m^1 = 1.5$ the worst possible payoff for player 1 when player 2 precommits to randomizes 50-50 (i.e., the mixed NE). Since the game is symmetric, the same is true for player 2.

Now suppose that the game is infinitely repeated

f) *Player 1 is a long-run player with discount factor δ; player 2 is a short-run player with discount factor 0. Find the set of perfect public equilibrium payoffs to the long-run player as a function of her discount factor.*

The minmax to player 1 delivers the same payoff of 1.5 than the worst possible NE (the mixed one), then, the worst dynamic payoff $\underline{v}^1$ is 1.5

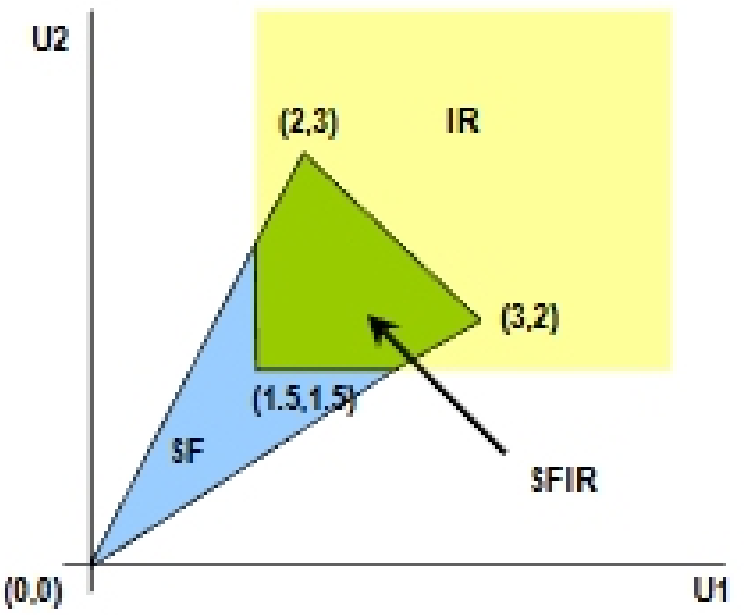
Both pure and mixed Stackelberg equilibria give the same payoff of 3, then, the best dynamic payoff $\overline{v}^1$ is 3.

Hence, the set of dynamic equilibria is composed by all the payoffs between 1.5 and 3. Since these two extreme cases correspond to static NE, this is true for all discount factors.

g) *Find strategies that support the best equilibrium from part f.*

Play the static NE (H, G) every period, getting 3 always

h) *Player 1 and 2 are both long-run players with common discount factor δ. When close to 1, describe the set of perfect equilibrium payoffs to both players.*



i) *Find a discount factor and strategies for part h such that both players receive an equilibrium payoff of 2.5.*

A payoff of 2.5 can be achieved for all discount factors just by public randomizing 50-50 between the static NE given by (H, G) and (G, H)

2 Greenspan

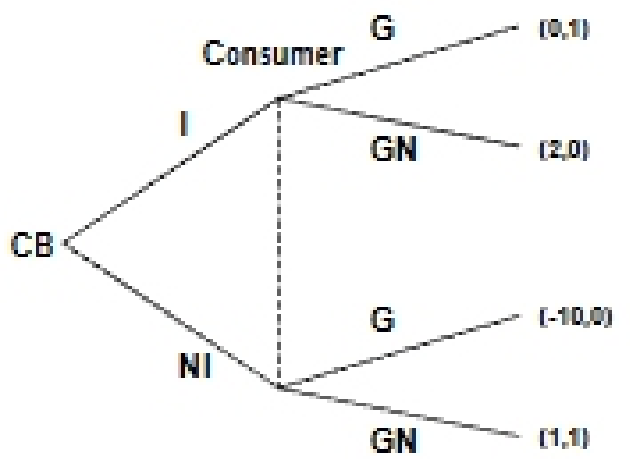
A long-lived central bank faces a short-run representative consumer. The bank must decide whether or not to inflate; the consumer must decide whether or not to expect inflation. If the consumer guesses correctly, she gets 1; incorrectly she gets 0. Central bank payoffs are

	Guess inflate (G)	Guess Not (GN)
Inflate (I)	0	2
Not inflate (NI)	-10	1

As a result of whether or not the central bank chose to inflate, economic activity is determined: there are two possibilities hyperinflation or price stability. If the bank chose to inflate the probability of hyperinflation is 1; if the bank chose not to inflate, the probability of hyperinflation is 10%. In all that follows, equilibrium *means perfect public equilibrium of the infinitely repeated game with public randomization.*

a) *Find the extensive and normal forms of the stage-game.*

The extensive form of the stage-game is,



and the stage game in normal form is,

	G	GN
I	0, 1	2, 0
NI	-10, 0	1, 1

b) *For the long-run player, find the minmax, the static Nash, mixed precommitment and pure precommitment payoffs.*