Math 231/EL1 Final

UIUC, May 7, 2013

Q.	Pt	Score	Q.	Pt	Score	Q.	Pt	Score	Q.	Pt	Score
1	16		6	22		11	8		16	9	
2	16		7	22		12	10		17	4	
3	16		8	5		13	20		18	5	
4	5		9	3		14	10		TA	2	
5	7		10	8		15	12		ExCr	10	
Tot.	60		Tot.	60		Tot.	60		Tot.	30	

1. (*8 points each*) Evaluate the integral.

   (a) $\int x \sin(3x)\, dx$

   (b) $\int \sec^4(5x)\, dx$

2. (*8 points each*) Evaluate the integral.

   (a) $\int \frac{3\cos^5 \alpha}{\sqrt{\sin \alpha}}\, d\alpha$

   (b) $\int \frac{dx}{(25+x^2)^{\frac{3}{2}}}$

3. (*8 points each*) Evaluate the integral.

   (a) $\int \frac{x^2}{x^2+4}$

   (b) $\int \frac{x+a}{x^2-x}\, dx$

4. (*5 points*) Determine whether the integral is convergent or divergent. If it is convergent, evaluate it.

   $\int_{-1}^{2} \frac{dx}{x^{11}}$

5. (*7 points*) Determine whether the integral is convergent or divergent. If it is convergent, evaluate it.

$$\int_0^\infty xe^{-3x}\,dx$$

6. (*11 points each*)

Determine if the series is **absolutely convergent**, **conditionally convergent** or **divergent**. Be sure to show your reasoning. No work, no credit.

(a) $\displaystyle\sum_{n=5}^{\infty} \frac{1}{\sqrt{n^3+30n}}$

(b) $\sum_{n=2}^{\infty}(-1)^n\frac{n+3}{n}$

7. (*11 points each*)

Determine if the series is **absolutely convergent**, **conditionally convergent** or **divergent**. Be sure to show your reasoning. No work, no credit.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{7^n}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln(n^n)}$

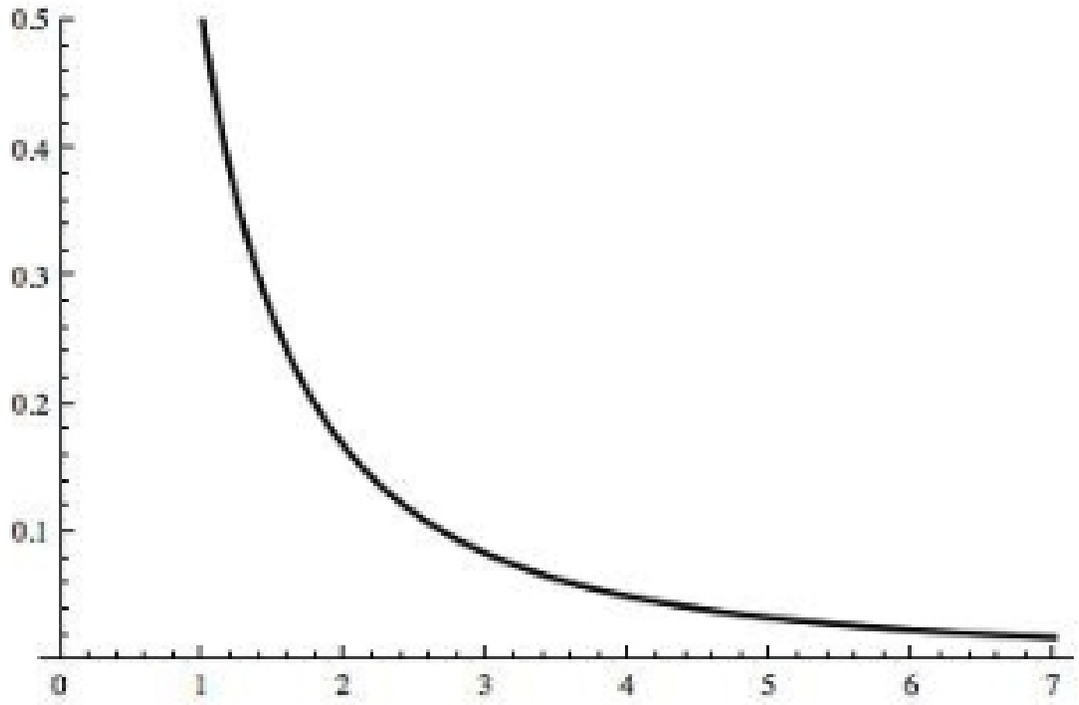
8. (*5 points*) Show that for any number $r \neq 1$ and positive integer k,

$$1+r+r^2+\cdots+r^k = \frac{1-r^{k+1}}{1-r}$$

9. (*3 points*) Draw on the diagram and give a brief explanation why

$$\sum_{n=2}^{6} \frac{1}{n(n+1)} \leq \int_1^6 \frac{dx}{x(x+1)}$$

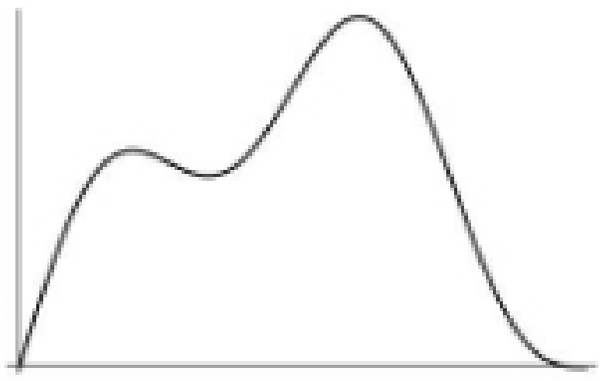
$$f(x) = \frac{1}{x(x+1)}$$



10. (*8 points, 2/3/3*)

This problem concerns the curve

$$y = 2\sin x + \sin 2x, \qquad 0 \le x \le \pi$$



(a) Give an integral for the length of the curve. You do not need to evaluate the integral.

(b) Give an integral for the area of the surface obtained by rotating the curve about the x-axis. You do not need to evaluate the integral.

(c) Give an integral for the area of the surface obtained by rotating the curve about the y-axis. You do not need to evaluate the integral.

11. (*8 points*) Short answer.

(a) Suppose that $c(x) = \sum_{n=0}^{\infty} c_n x^n$ converges for $x = -4$ but diverges for $x = 6$.

i. $\sum_{n=0}^{\infty}(-1)^n c_n$ (absolutely converges/ conditionally converges/diverges).

ii. $\sum_{n=0}^{\infty}(-7)^n c_n$ (absolutely converges/ conditionally converges/diverges).