

**Discrete Structures
(COT 3100) Final Exam
Spring 2000
Section 2
4/25/00**

Lecturer: Arup Guha

TA: _____

First Name : _____

Last Name : _____

(Note: On all questions that contain a blank line, please place your answer on that line clearly. If a short answer question doesn't have a line for you to place your answer, please neatly write your answer directly below the question. Circle TRUE or FALSE for all true/false questions.)

1) (15 pts) True/False. You will get one point for each question answered correctly, zero points for each question left blank, and negative one point for each incorrect response.

- | | | |
|--|-------------|--------------|
| a) $(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$ | TRUE | FALSE |
| b) $\exists x \exists y [x + y = 10]$ | TRUE | FALSE |
| c) If the contrapositive of a statement is true, then the converse of statement is true as well. | TRUE | FALSE |
| d) $\neg[\neg \exists x p(x)] \rightarrow \exists x \neg p(x)$ | TRUE | FALSE |
| e) Let A, B and C be sets.
If $A \subseteq B$ and $A \cap B = B \cap C$, then $A = C$. | TRUE | FALSE |
| f) The total number of subsets of $\{2,4,5,6,9\}$ is 25. | TRUE | FALSE |
| g) If $a \mid b$ and $a \mid c$, then $\gcd(b,c) \mid a$. | TRUE | FALSE |
| h) If R is a bijection ($A \rightarrow B$), and S is a surjection ($B \rightarrow C$), then $R \circ S$ is an injection ($A \rightarrow C$). | TRUE | FALSE |
| i) Let A and B be sets of strings such that $A^* \subseteq B^*$, then $A \subseteq B$. | TRUE | FALSE |
| j) The number of bijective functions over a set $A = \{1,2,3\}$ is 6. | TRUE | FALSE |
| k) A DFA must have more than one final state. | TRUE | FALSE |
| l) A language containing a finite number of strings is a regular language. | TRUE | FALSE |
| m) A DFA with n states that accepts at least one string MUST accept at least one string with a length less than or equal to n. | TRUE | FALSE |
| n) a^+ is NOT a valid regular expression. | TRUE | FALSE |
| o) $ \emptyset = 0$. | TRUE | FALSE |

2) (8 pts) Fill in the following Truth Table to evaluate the expression $(p \rightarrow (q \rightarrow \neg r)) \rightarrow (r \rightarrow \neg q)$.

p	q	r	$q \rightarrow \neg r$	$p \rightarrow (q \rightarrow \neg r)$	$r \rightarrow \neg q$	$(p \rightarrow (q \rightarrow \neg r)) \rightarrow (r \rightarrow \neg q)$
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

3) Let the set $A = \{1, 4, 9, 16, 25, 36, 49, 64\}$.

a) (4 pts) How many non-empty subsets of A have the property that the product of all of their elements is less than or equal to 100? _____

b) (2 pts) How many subsets of A contain no odd numbers? _____

c) (2 pts) How many subsets of A contain at least one odd number? _____

4) (4 pts) Use Euclid's Algorithm to find the greatest common divisor of 240 and 87.

5) (5 pts) Draw a DFA to recognize the language of all binary strings (over the alphabet 0 and 1) that are divisible by 4.