251y0341 05/05/03

ECO 251 QBA1
FINAL EXAM
MAY 7, 2003

Name _KEY___________
Class ________________

Part I. Do all the Following (14 Points) **Make Diagrams! Show your work!**
$x \sim N(3, 4)$. How many of you really believe that a probability can be negative?

1. $P(-2.30 \le x \le 1.85) = P\left(\frac{-2.30 - 3}{4} \le z \le \frac{1.85 - 3}{4} \right) = P(-1.32 \le z \le -0.29)$

$= P(-1.32 \ge z \le 0) - P(-0.29 \le z \le 0) = .4066 - .1141 = .2925$

or $= P(-1.33 \ge z \le 0) - P(-0.29 \le z \le 0) = ..4082 - .1141 = .2941$

or , better, $= P(-1.325 \ge z \le 0) - P(-0.29 \le z \le 0) = .4074 - .1141 = .2933$

2. $P(1 \le x \le 17) = P\left(\frac{1 - 3}{4} \le z \le \frac{17 - 3}{4} \right) = P(-0.50 \le z \le 3.50)$

$= P(-0.50 \ge z \le 0) + P(0 \le z \le 3.50) = .1915 + .4998 = .6913$

3. $P(x \ge 1.85) = P\left(z \ge \frac{1.85 - 3}{4} \right) = P(z \ge -0.29) = P(-0.29 \le z \le 0) + P(z \ge 0)$

$= .1141 + .5 = .6141$

4. $F(4.00)$ (Cumulative Probability) $= P(x \le 4) = P\left(z \le \frac{4 - 3}{4} \right)$

$= P(z \le 0.25) = P(z \ge 0) + P(0 \le z \le 0.25) = .5 + .0987 = .5987$

5. $P(-4.00 \le x \ge 4.00) = P\left(\frac{-4 - 3}{4} \le z \le \frac{4 - 3}{4} \right) = P(-1.75 \le z \le 0.25)$

$= P(-1.75 \le z \le 0) + P(0 \le z \le 0.25) = .4599 + .0987 = .5586$

6. $x_{.015}$ (Find $z_{.015}$ first) **Make a diagram** for z. Show a Normal curve with a mean of zero in its center. Remember that $z_{.015}$ is a point with 1.5% above it and 98.5% below it. Since 50% of the distribution is bellow zero $P(0 \le z \le z_{.015}) = .9850 - .5 = .4850$. According to the Normal table $P(0 \le z \le 2.17) = .4850$. So $z_{.015} = 2.17$ and
$x = \mu + z_{.015}\sigma = 3 + (2.17)4 = 3 + 8.68 = 11.68$.
Check: $P(x \ge 11.68)$
$= P\left(z \ge \frac{11.68 - 3}{4} \right) = P(z \ge 2.17) = P(z \ge 0) - P(0 \le z \ge 2.17) = .5 - .4850 = .0150$

7. A symmetrical region around the mean with a probability of 25%. **Make a diagram** for z. Show a Normal curve with a mean of zero in its center. If we split 25% in two, we get two areas, one on either side of the mean with probabilities of 12.5%. We can call the point we want $z_{.375}$, because, since the area above zero is 50%, the area above $z_{.375}$ must be 50% - 12.5% = 37.5%. But we have already decided that the probability between $z_{.375}$ and zero is 12.5%. The closest we can come on the Normal table is $P(0 \le z \le 0.32) = .1255$, so $z_{.375} \approx 0.32$ and $x = \mu \pm z_{.375}\sigma = 3 \pm (0.32)4 = 3 \pm 1.28$ or 1.72 to 4.28,

Check:

$$P(1.72 \le x \le 4.28) = P\left(\frac{1.72 - 3}{4} \le z \le \frac{4.28 - 3}{4}\right) = P(-0.32 \le z \le 0.32) = 2P(0 \le z \le 0.32) = 2(.1255)$$

$$= .2510$$

Exam is normed on 75 points. There are actually 128 possible points.

II. (10 points+-2 point penalty for not trying part a .) **Show your work!**
The following numbers apply to 9 developed countries and give deaths per 100 million miles and speed limits. (xy was not supplied and is calculated in red.)

Row	deaths x	SpLim y	xy
1	3.1	55	170.5
2	3.4	55	187.0
3	3.5	55	192.5
4	3.6	70	252.0
5	4.2	55	231.0
6	4.4	60	264.0
7	4.8	55	264.0
8	5.0	60	300.0
9	6.2	75	465.0
			2326.0

These sums have been calculated for you. $\sum x = 38.2$, $\sum x^2 = 169.86$, $\sum y = 540$ and $\sum y^2 = 32850$. Please calculate the following:

a. The sample standard deviation of x (4) Note that $\bar{y} = 60.00$ and $s_y = 7.50$.

Many of you wasted time and energy computing the x squared and y squared columns and then decided that you could get the xy sum by multiplying the sums of x and y. Where have you been?

b. The sample covariance between x and y. (3)

c. The sample correlation between x and y. (2)

d. Given the size and sign of the correlation, what conclusion might you draw on the relation between speed and safety if this were the only evidence available? (1)

e. Assume that the death rate in all 9 countries fell by .1. What would be the new values of $\bar{x}$, s_x, s_{xy} and r_{xy}. Use only the values you computed in a-c and rules for functions of x and y to get your results. If you state the results without explaining why, or change x and recompute the results, you will receive no credit. (4). How many of you recomputed the results anyway?

Solution: a) $\bar{x} = \frac{\sum x}{n} = \frac{38.2}{9} = 4.24444$

$$s_x^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \frac{169.86 - 9(4.24444)^2}{8} = \frac{7.72256}{8} = 0.9653 \quad s_x = \sqrt{0.9653} = 0.9825.$$

$$\bar{y} = \frac{\sum y}{n} = \frac{540}{9} = 60.00 \qquad s_y^2 = \frac{\sum y^2 - n\bar{y}^2}{n-1} = \frac{32850 - 9(60.00)^2}{8} = \frac{450}{8} = 56.25$$

$s_y = \sqrt{56.25} = 7.50$

b) We found above that $\sum xy = 2326$, so

$$s_{xy} = \frac{\sum xy - n\bar{x}\bar{y}^2}{n-1} = \frac{2326 - 9(4.24444)(60.00)}{8} = \frac{34}{8} = 4.25$$

c) $r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{4.25}{(0.9825)(7.50)} = .5768$ This must be between -1 and 1.

d) If we square the correlation we get 0. 333, which in a zero to one scale is not impressive. I would not want to conclude that speed limits and safety are closely related, though speed limits may be a factor.