Assignment 1

COT 3100 ; Summer 2021

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1) (10 pts) Use logical identities to show that $\neg p \to (q \to r) \equiv q \to (p \vee r)$. To receive full credit, you must show the transformations step by step so the reader can follow your reasoning, and you must identify each logical identity that is used in the sequence of transformations.

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2) (15 pts) Negate the following Boolean expression and simplify the result as much as possible. Please show each step and name the rule you are using at each step:
$p \vee q \vee (\bar{p} \wedge \bar{q} \wedge r)$

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3) (10 pts) Use the truth table method to prove the following two expressions are logically equivalent:

- $(p \vee q) \to r$
- $(p \to r) \wedge (q \to r)$

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4) (15 pts) Use the laws of logic to show that two following expressions are logically equivalent:

- $(p \wedge (\bar{p} \to r)) \vee ((q \wedge s) \vee (q \wedge \bar{s}))$
- $p \vee q$

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5) (20 pts) Use the rules of inference to prove the following argument:

$p \vee q$
$p \to r$
$q \to r$
$s \vee t$
$\bar{t}$

$\therefore r \wedge s$

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6) (20 pts) Let "?" be an unknown Boolean logical operator. The logical statement $[(\bar{p} \wedge q) \vee r] \to (q ? r)$ is equivalent to $(p \vee \bar{q} \vee r)$. Given this information, there are 2 possible truth tables for the Boolean logical operator "?". List, with proof, both of these truth tables.

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7) (10 pts) Find your own open numerical statement $P(x, y)$ (with the universe of positive integers) such that exactly one of $\forall x(\exists y | P(x, y))$ and $\exists x(\forall y | P(x, y))$ is true.