CPS102- Homework 1

Due on September 15, 2005

Questions may continue on the back. Please write clearly. What I cannot read, I will not grade. Typed homework is preferable. A good compromise is to type the words and write the math by hand.

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I have adhered to the Duke Community Standard in completing this assignment.

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In all answers, show your work in detail. The first two questions are warm-up problems from the book (supplementary exercises 2 and 6 on pages 114-115).

1. Find the truth table of the following compound proposition:

$$(p \vee q) \rightarrow (p \wedge \neg r)$$

2. Show that these statements are inconsistent: "If Sergei takes the job offer then he will get a signing bonus." "If Sergei takes the job offer, then he will receive a high salary." "If Sergei gets a signing bonus, then he will not receive a high salary." " Sergei takes the job offer."

You may use truth tables or inference rules. Either way, start by transforming the propositions above into appropriate symbolic form.

3. In class and in the book you saw several truth tables that take two propositions as their input. The operators associated to these tables are called *binary*, because they take two propositions as operands. For instance,

p	q	
T	T	T
T	F	F
F	T	F
F	F	F

is the truth table for the conjunction ("and") of the two propositions p and q, that is, the truth table of $p \wedge q$, so conjunction is a binary operator. How many distinct binary truth tables could one conceivably make up? The answer is sixteen: Truth tables for binary operators all have four rows, and the tables differ from one another by the content of the four elements in the last column. There are sixteen possible ways to fill the last column, and here they are:

p	q	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	r
T	T	T	T	T	T	T	T	T	T	F	F	F	F	F	F	F	F
T	F	T	T	T	T	F	F	F	F	T	T	T	T	F	F	F	F
F	T	T	T	F	F	T	T	F	F	T	T	F	F	T	T	F	F
F	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F

Some of these are familiar: `a` is a tautology, `r` is a contradiction, `b` is $p \vee q$, `c` is $q \rightarrow p$, `e` is $p \rightarrow q$, and so forth. Some others we have never seen before.

Table `i` is called the *nand* of p and q. Let us use a special symbol for it, $p \mid q$ (called *Sheffer's stroke*). Let us repeat its truth table here:

p	q	$p \mid q$
T	T	F
T	F	T
F	T	T
F	F	T

.

From this table, we can see immediately the following logical equivalence:

$$p \mid q \equiv \neg(p \wedge q) ,$$

so we can obtain the table for "and" (column `h`) from that of "nand" by inverting the equivalence:

$$p \wedge q \equiv \neg(p \,|\, q) \;.$$

Write sixteen lines in the following format:

$$\texttt{x} : \; \textit{expression}$$

where `x` represents one of the sixteen letters in the column headings of the table above, and *expression* is an implementation of the corresponding truth table using nothing other than the symbols p, q, "nand" operators and parentheses. For example,

$$\texttt{i} : \; p \,|\, q \;.$$

There may be different answers for each column. Just give any one. Show your reasoning, and also give the final table neatly written with the columns in alphabetical order. [Hint: Column `m` is the truth table for $\neg p$, which can be implemented as follows:

$$\texttt{m} : \; p \,|\, p$$

(check with a two-row truth table that this is the case). It is best to build your answer from the easiest cases, and derive harder answers from easier ones.]

4. Express the following sentence in predicate logic:

Nobody is right all the time but everybody is right some of the time.

In doing so, use the predicate $R(x, t)$ meaning "person x is right at time t."

5. If we combine supplementary problems 14 and 16 on page 115 of the textbook, we are asked to show that the implication

$$\exists x \,\forall y \; P(x, y) \;\rightarrow\; \forall y \,\exists x \; P(x, y) \tag{1}$$

is a tautology, while the converse implication

$$\forall y \,\exists x \; P(x, y) \;\rightarrow\; \exists x \,\forall y \; P(x, y) \tag{2}$$

is not. Rather than doing this in general, it may be instructive to show this result in the special case in which the domain (or universe of discourse) for x and y is finite. This allows transforming each existential quantifier to the "or" of a finite number of propositions, and each universal quantifier to the "and" of a finite number of propositions, as shown in the text.

Let us do this. For the following questions, assume that the universe of discourse for x is the set $\{x_1, x_2\}$, and that the universe of discourse for y is the set $\{y_1, y_2\}$.

(a) Rewrite the two propositions

$$\exists x \,\forall y \quad P(x, y) \tag{3}$$
$$\forall y \,\exists x \quad P(x, y) \tag{4}$$

using only "and" and "or" operators, and parentheses as needed. State which is which.

(b) Transform one of the two resulting expressions so that each of them is the disjunction of several conjunctions (the "or' of several "and"s).

(c) Use a simple argument to show that (1) is a tautology and (2) is not.