EE 428 **EXAM I** **11 October 2005**

Last Name (Print): Solutions

First Name (Print): ____________________

ID number (Last 4 digits): __________

Section: ___

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Problem	Weight	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

INSTRUCTIONS

1. You have 2 hours to complete this exam.
2. This is a closed book exam. You may use one 8.5" × 11" note sheet.
3. Calculators are allowed.
4. Solve each part of the problem in the space following the question. If you need more space, continue your solution on the reverse side labeling the page with the question number; for example, **Problem 1.2 Continued**. **NO** credit will be given to solutions that do not meet this requirement.
5. **DO NOT REMOVE ANY PAGES FROM THIS EXAM.** Loose papers will not be accepted and a grade of **ZERO** will be assigned.
6. The quality of your analysis and evaluation is as important as your answers. Your reasoning must be precise and clear; your complete English sentences should convey what you are doing. **To receive credit, you must show your work.**

Problem 1: (25 Points)

1. (10 points) A SISO LTI system with input $u(t)$ and output $y(t)$ is represented by the transfer function

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{s^4 + 1}{s^4 + 4\,s^2 + 2\,s + 9}.$$

Using the smallest number of integrators, construct an all-integrator block diagram representation of the system, and then provide a state -space representation

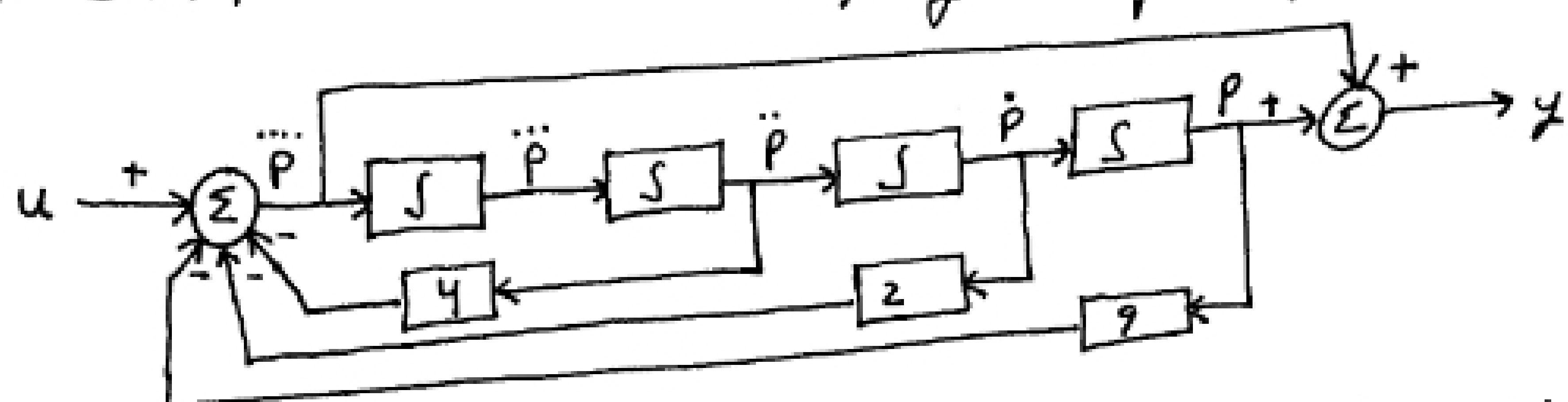
$$\begin{aligned} \dot{x} &= Fx + Gu \\ y &= Hx + Ju. \end{aligned}$$

Clearly indicate the assignment of state-variables in the all-integrator block diagram.

$$\frac{Y}{U} = \frac{P}{U}\frac{Y}{P} = \left(\frac{1}{s^4 + 4s^2 + 2s + 9}\right)\left(s^4 + 1\right)$$

$$s^4 P + 4s^2 P + 2s\,P + 9P = u \Rightarrow \ddddot{p} = -4\ddot{p} - 2\dot{p} - 9p + u$$

$$Y = s^4 P + P \qquad \Rightarrow y = \ddddot{p} + p$$



Choose $x_1 = p$, $x_2 = \dot{p}$, $x_3 = \ddot{p}$, and $x_4 = \dddot{p}$ to obtain

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -9 & -2 & -4 & 0 \end{pmatrix}}_{=F} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}}_{=G} u$$

$$y = \dot{x}_4 + x_1 = (-4x_3 - 2x_2 - 9x_1 + u) + x_1$$

$$y = \underbrace{(-8,\ -2,\ -4,\ 0)}_{=H} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \underbrace{(1)}_{=J} u$$

2. (15 points) Consider the closed-loop system in Figure 1 with reference input $R(s)$, disturbance input $W(s)$, and controlled output $Y(s)$. The PI controller gains K and K_I are free parameters for the control engineer to select.

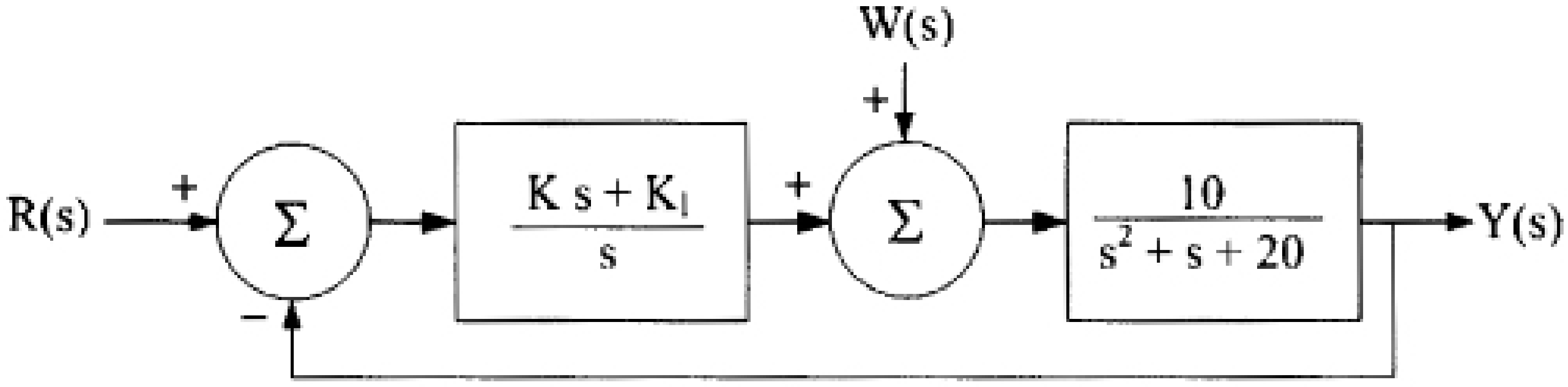
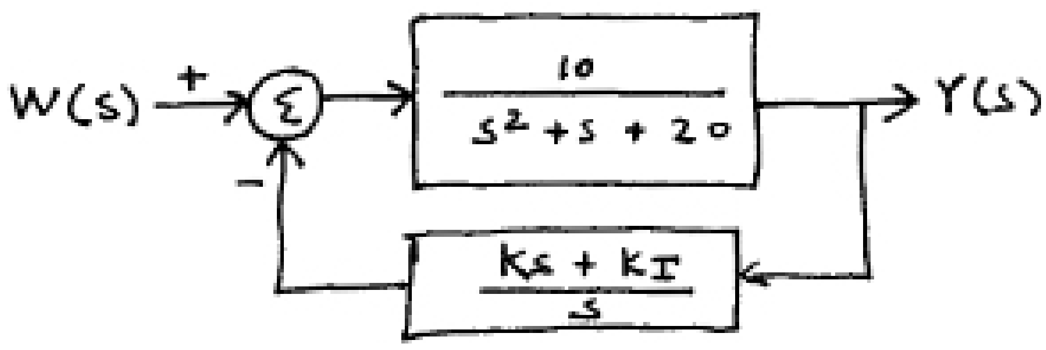


Figure 1: Closed-loop system with PI control gains K and K_I.

(a) (5 points) Determine the transfer function from the disturbance input to the controlled output and express your answer in the standard form

$$\frac{Y(s)}{W(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}.$$



$$\frac{Y}{W} = \frac{\frac{10}{s^2+s+20}}{1 + \frac{10}{s^2+s+20}\,\frac{Ks+K_I}{s}}$$

$$\boxed{\frac{Y}{W} = \frac{10s}{s^3 + s^2 + (20+10K)s + 10K_I}}$$

(b) (10 points) What is the system type and error constants (K_p, K_v, and K_a) with respect to disturbance rejection ?

First find $\frac{E(s)}{W(s)}$ for $R(s) = 0$. $E(s) = \cancel{R(s)}^{\,0} - Y(s) = -\frac{Y(s)}{W(s)}\,W(s)$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} sW(s)\,\frac{-10s}{s^3+s^2+(20+10K)s+10K_I}$$

Note that for $W(s) = \frac{1}{s}$ and $W(s) = \frac{1}{s^2}$, e_{ss} is finite; but for $W(s) = \frac{1}{s^3}$, $|e_{ss}| \to \infty$. And so with respect to the disturbance input, the system is Type 1