

## Review for Exam I

Exam I will cover some of the following topics and types of problems.

1. Be sure you know and are able to apply **all** of the following definitions **very well**. Where applicable you should be able to write the definition in **both** words and mathematical symbols.
  - Verbal, listing, and set builder descriptions of sets
  - Unions, complements, intersections of sets
  - Subset, disjoint sets, empty set
  - One to one correspondence
  - Finite set; Infinite set
  - Whole numbers; natural numbers
  - Definition of multiplication as repeated addition as printed in notes and text
  - Definition of division as repeated subtraction as printed in notes and text
  - Cartesian product
2. Know the names and be able to apply the properties of whole number addition and multiplication (pages 108 and 126)
3. Know all of the following models. Be able to state the definition(s), give examples, and solve problems *strictly* following the models.
  - Set model of addition
  - Missing addend model and missing factor model
  - Number line models for addition and subtraction. (Be careful with arrows!)
  - Multiplication tree model.
  - Cartesian product model (you must use the definition of the Cartesian product of two sets for this model!)
  - Repeated subtraction model for division.
4. Know how to write a number in base 5 as a number in base 10 and vice-versa.
5. Know how to add or subtract numbers in base 5.
6. Know the following algorithms, be able to compute with them, and explain the steps.
  - Addition and subtraction with manipulatives, place value cards, place value diagrams, instructional diagrams, and the “final algorithm”.
  - Division with repeated subtraction.
  - Multiplication of small numbers with place value cards
  - Multiplication with expanded notation in the instructional algorithm.
  - Scaffold method

Example Problems:

1. (a) Let  $A$  and  $B$  be sets. Give a verbal and a mathematical definition for  $A \cup B$ ,  $A \cap B$ ,  $\bar{A}$ ,  $\emptyset$   
 (b) Let  $U = \{x | x \text{ is a letter in the English alphabet or } x \text{ is a whole number less than } 5\}$  be the universe,  $A = \{0, 1, 2\}$ ,  $B = \{a, b, c, d, e\}$ ,  $C = \{0\}$ . Find  $\bar{A}$ ,  $A \cup B$ ,  $A \cap B \cap C$ ,  $\emptyset \cap C$ ,  $C \cup \emptyset$ .
2. (a) Find a one-to-one correspondence which shows that the set of even numbers is equivalent to the set of odd numbers.

- (b) If two sets are equal, are they also equivalent? Explain if this is true. If it is not true, give an example which shows that the statement is not true.
- (c) If two sets are equivalent are they also equal? Explain if this is true. If it is not true, give an example which shows that the statement is not true.
3. (a) Your classmate claims that  $\{0\} = \emptyset$ . Explain why is she incorrect and how you would address her misunderstanding.
- (b) Provide examples of: two sets which are disjoint and two sets where one set is a proper subset of the other.
- (c) If  $A \subset B$  and  $B \subset C$  is  $A \subset C$ ? Use Venn diagrams to explain your answer. Provide a practical example which demonstrates this concept.
4. There are 100 senators on student government and there are three optional committees academics, Greek life, and social events. Suppose 30 senators are on academics, 40 are on Greek life, and 30 are on social events. Suppose also that 10 senators serve on both academics and Greek life, 10 on both academics and social events, and 15 on both Greek life and social events. There are 5 senators on all three. How many senators are on more than one committee and how many are on no committees? You must show your work.
5. (a) Demonstrate the addition  $254 + 299$  with blocks, strips, and mats. Indicate all exchanges that you make.
- (b) Demonstrate the addition  $254 + 299$  with place value diagrams and with the final algorithm. Indicate all exchanges that you make.
- (c) Consider the problem:

$$\begin{array}{r} 1 \\ 29 \\ +15 \\ \hline 44 \end{array}$$

Explain why it is misleading to say that we “carry the one” when we add the 9 and 5 digit. Describe this step in a more conceptually relevant way.

6. (a) Explain the missing factor model of division.
- (b) Show a 3rd grade student how to solve  $36 \div 4$  with missing factor. You can assume that the 3rd grader has a multiplication table but *does not know anything* about algebra. You will lose points if you write something the 3rd grader won’t understand.
- (c) Use missing factor to explain to a group of older students why  $a \div 0$  is not defined. (Hint: consider two cases and remember that the  $c$  in missing factor is a unique number).
7. (a) Convert 42 to base five.
- (b) Convert  $(112)_{five}$  to base ten.
- (c) Compute  $(312)_{five} + (333)_{five}$  in base five.
8. (a) Compute  $5 \cdot 213$  using place value cards. Indicate all exchanges that you make.
- (b) Compute  $17 \cdot 223$  using expanded notation.
- (c) Compute  $17 \cdot 223$  in the instructional algorithm.
- (d) Explain the relationship between the computations in (b) and (c).
9. (a) Use repeated subtraction (taking many multiples at a time) to compute  $12,451 \div 7$ .
- (b) Use the scaffold method to compute  $12,451 \div 7$ .
- (c) Explain the relationship between the computations in (a) and (b).
10. Below you have an arithmetic problem and a model. Come up with an appropriate word problem for the problem and solve it using the model. Draw diagrams and show lots of work.

- (a)  $4 \cdot 5$  in the multiplication tree model.
- (b)  $3 \cdot 4$  in the Cartesian product model.
- (c)  $27 \div 3$  in the repeated subtraction model.