

1 Introduction

- To provide understanding of the requirements for a computational model of a dynamic system

1.1 Objective(s)

1. Understand mathematical models of simple systems that represent a large number of real-world systems.
2. Apply Matlab® to understanding how these systems behave to various inputs.
3. Understand how a system's response to various inputs is used to determine system parameters, mass, stiffness, damping, inertia, etc.

1.2 Learning Outcomes

At the end of performing this laboratory exercise the student will be able to:

- Use Matlab® to analyze the response of various systems to various inputs.

2 Dynamic Models

2.1 Translational Motion - Cruise Control (Example 2.1, Franklin)

2.1.1 Objectives

1. Write the equations of motion for the forward speed and position of the automobile.
2. Take the Laplace transform of the resulting differential equation and solve for the transfer function between the input and output.
3. Use Matlab® to find the response of the velocity of the automobile to a step input at $t=0$ of $u=500\text{N}$. Parameters for the simulation: $m=1000\text{kg}$, viscous drag coefficient $b=50\text{ N sec/m}$.

2.1.2 Assumptions

1. Neglect rotational inertia of wheels, etc.
2. All dissipative forces, (friction, viscous, drag, etc.) represented as being proportional to the automobiles speed.

2.1.3 Solution

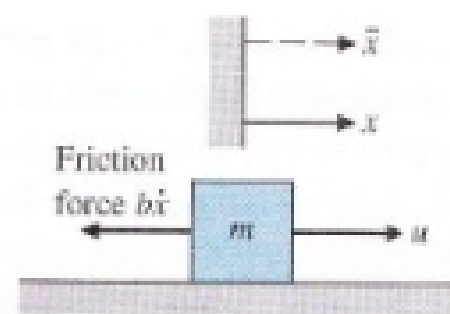


Figure 1, Free-body Diagram of Automobile Cruise Control Problem [1]

Sum forces $\sum F = ma = m \frac{d^2x(t)}{dt^2} = m\ddot{x}$ (1)

$$u - b\dot{x} = m\ddot{x} \quad (2)$$

Rearrange $\ddot{x} + \frac{b}{m}\dot{x} = \frac{u}{m}$ (3)

Convert from position to velocity $\dot{v} + \frac{b}{m}v = \frac{u}{m}$ (4)

Take the Laplace Transform $sV(s) + \frac{b}{m}V(s) = \frac{1}{m}U(s)$ (5)

Rearrange $\frac{V(s)}{U(s)} = \frac{1/m}{s + b/m}$ (6)

Plot the step response using Matlab®

$$\text{num} = \frac{1}{m} = \frac{1}{1000}$$

$$\text{den} = \left[1 \quad \frac{b}{m} \right] = \left[1 \quad \frac{50}{1000} \right]$$

Table 1, Matlab® Code (command-line) to Produce Step Response of Equation (6)

```

num = 1/1000;           % 1/m
den = [1 50/1000];     % s + b/m
sys = tf(num*500, den); % step produces unit step response,
step(sys);             % num*500, u=500

```

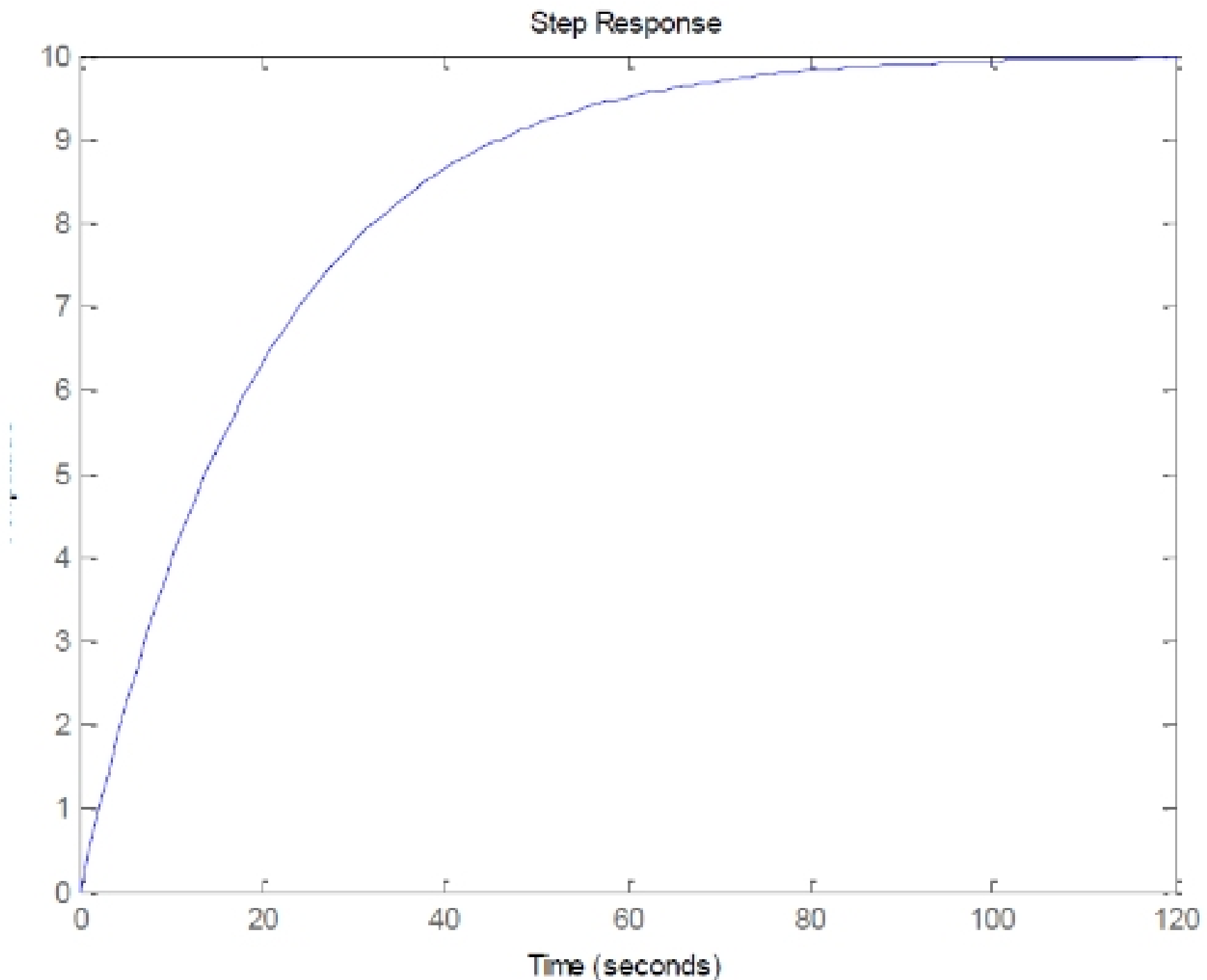


Figure 2, Response of Automobile Velocity to Step Response of 500 N

2.2 Rotational Motion - DC Motor (Example 2.13,3.17, Franklin)

2.2.1 Objectives

1. Write the equations of motion for a DC Motor with its electrical circuit, (velocity and position).
2. Take the Laplace transform of the resulting differential equation and solve for the transfer function between the input and output.
3. Use Matlab® to analyze the response of the velocity of the DC motor to a step input.

2.2.2 Assumptions

- The system has an inertia I_m and viscous friction coefficient b .

2.2.3 Solution