AAE 340

Dynamics and Vibrations

Exam I

Instructions:

The exam is worth a total of 100 points. Work as quickly and accurately as you can.

Write your name at the bottom of this page.

DO NOT TURN THE PAGE UNTIL

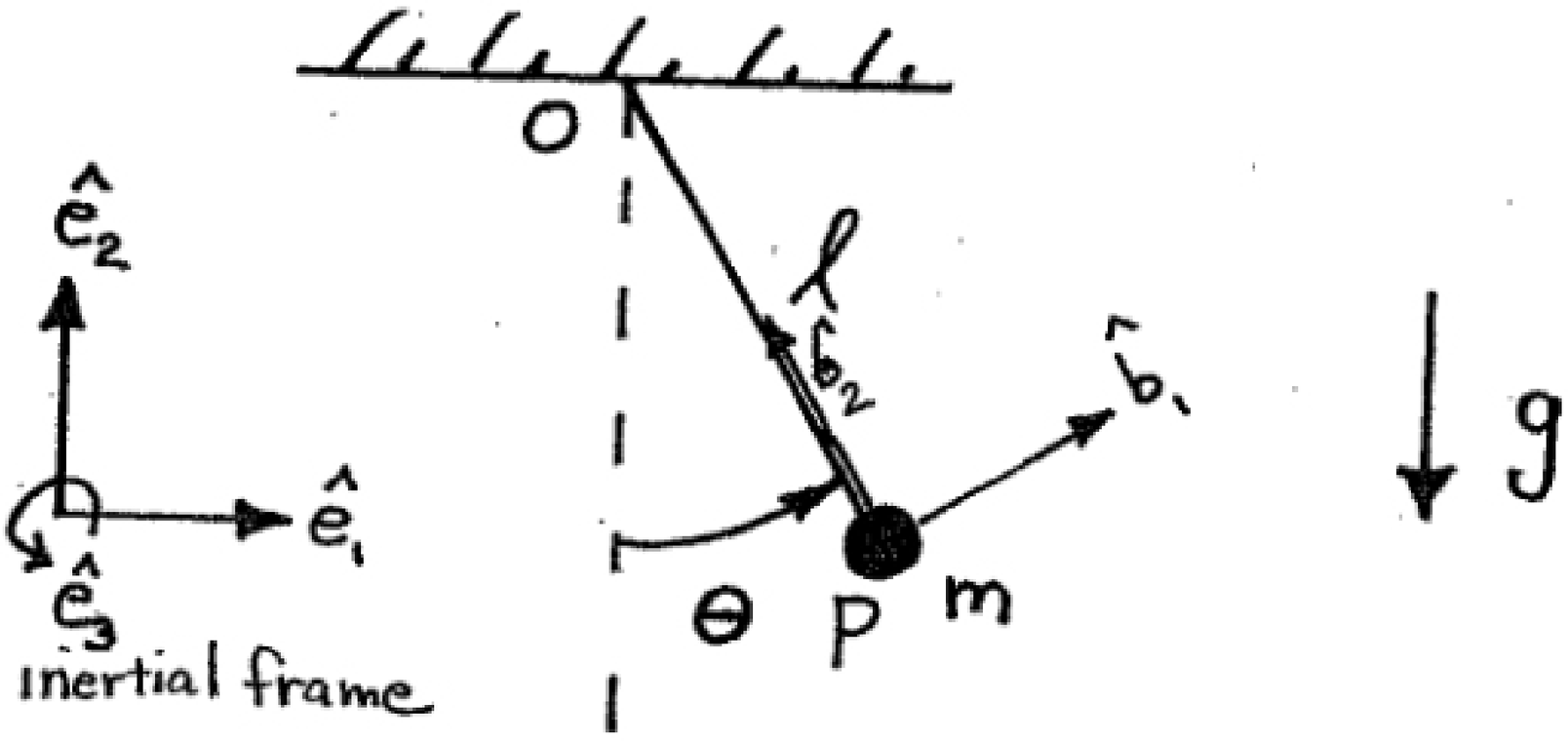
INSTRUCTED TO DO SO

Note: Partial Credit can only be given for

1. **Correct** partial steps toward the complete solution, which are
2. Clearly labeled in a logical and **systematic** manner.

Name _____

I. Pendulum Problem (35 points)



The simple pendulum is restricted to motion in the inertial plane, $\hat{e}_1 - \hat{e}_2$. Assume: particle P has a mass of m, the rod is massless and of length ℓ, the pivot point at O is frictionless, and the gravity, g, is uniform.

Ia. Derive the EOM in terms of θ.

For full credit, label the four steps discussed in the lecture. Use the BKE to write the acceleration of point P in b coordinates.

Ib. Give an equation for the constraint force (a vector) on P.

II. Analysis of an Equation of Motion (35 points)

Consider the following equation of motion:

$$\ddot{\theta} + \frac{g}{\ell}\sin\theta = 0$$

IIa. Put the EOM in state variable form.

IIb. Assume θ is a small angle so that $\sin\theta \approx \theta$. Use this approximation to simplify your EOM. Divide out any unnecessary constants and put your EOM in the simplest possible form.

IIc. Solve the d.e. obtained in **IIb.** in the form

$$\theta(t) = C\cos(\omega_n t + \phi)$$

Derive an expression for ω_n (the natural frequency) in terms of g and ℓ. Find the constants C and ϕ in terms of the initial conditions $\theta_0 = \theta(0)$, $\dot{\theta}_0 = \dot{\theta}(0)$.

IId. Give an expression for the period of the solution obtained in **IIc.** in terms of g and ℓ.