StudyGuide Test 1

Advanced Calculus

Definitions to Memorize

Least Upper Bound (Sup)

S is the least upper bound for set A if
(i) s is an upper bound for A
(ii) if b is any upper bound for A, then $s \leq b$.

Greatest Lower Bound(Inf)

I is the greatest upper bound for set A if
(i) I is an lower bound for A
(ii) if b is any lower bound for A, then $i \geq b$.

Coutable Set

if it can be put into 1-1 correspondence with $\mathbb{N}$

Sequence

A sequence is a function whose domain is $\mathbb{N}$

Subsequence

A subsection of a sequence with new indexes that also is a sequence (therefore infinite) - the order of terms has to be the same and repetition is not allowed. An example of which would be if the sequence is the natural numbers, then the subsequence could be the evens or the odds.

Subsequence Convergence

Subsequence of a convergent sequence converge to the same limit as the orginal sequence. This also allows us to say that if two subsequences of the same sequence converge to different limits the original sequence diverges.

Cauchy Sequence

(A_n) is Cauchy if $\forall \varepsilon > 0 \; \exists N \in \mathbb{N}$ st $\forall n,m \geq N$, $|A_n - A_m| < \varepsilon$

Also, every convergent sequence is cauchy and cauchy sequences are bounded.

Bounded Sequence

A sequence (X_n) --> is bounded if $\exists M > 0$ st $|X_n| \leq M \; \forall n \in \mathbb{N}$

Picture:

Monotone Sequence

A sequence is monotone if its either increasing or decreasing

Increasing or Decreasing sequence is: if $A_n \leq A_{n+1}$ ∀ n ϵ ℕ then it is increasing. if $A_n \geq A_{n+1}$ ∀ n ϵ ℕ then it is decreasing

Convergent Series

Let $\sum(B_n)$ be a infinate series. A corresponding sequence of partial sums is defined by $S_n = b_1 + b_2 + \ldots + b_n$. The series converges to where, the sequence (S_n) --> B, B.

NOTE: if S_n is monotone and bounded then the series $\sum(B_n)$ converges. (which makes sense cause MCT).

Sequence Converges

A sequence converges (A_n) converges to a if ∀ $\varepsilon > 0$, ∃ an N ϵ ℕ st ∀ n≥N, $|A_n - a| < \varepsilon$.
i.e. After the chopping point N, all of the terms are within ε distance of a

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Theorems to Memorize

Axiom of Compleatness

Every nonempty set of real numbers that is bounded above has a least upper bound

Nested Interval Property

If there is a sequcence of closed intervals, and they are nested (i.e. each interval is contained in the previous interval), then the intersection of these intervals is not empty.

Picture:

Lemma 1.3.7

Let S be an upper bound of A then s= supA iff ∀ ε> 0 ∃ a ϵ A st s-ε < a.

Discussion: If you subract a really small number then there is some a that is in the set that is bigger

Picture:

Monotone Covergence Theorem

If a sequence is a Monotone AND Bounded then it converges.

Bolzano-Weirerstrass Theorem

Every Bounded sequence contains a convergent subsequence.

Cauchy Criterion

A sequence converges if and only if it is a Cauchy sequence

Algebraic Limit Theorem

Let (A_n) -> a and (B_n) -> b
Then:

(i) (cA_n) -> ca $\forall c \in \mathbb{R}$
(ii) $(A_n$ +/- $B_n)$ -> a +/- b
(iii) $(A_n * B_n)$ -> a*b
(iv) (A_n / B_n) -> a/b, assuming $B_n \neq 0$ and $b \neq 0$

Order Limit Theorem

Let (A_n) -> a and (B_n) -> b
Then:

(i) if $A_n \geq 0 \; \forall n \in \mathbb{N}$ then it must be that $a \geq 0$
(ii) if $A_n \leq B_n \forall n \in \mathbb{N}$ then it must be that $a \leq b$
(iii) if $\exists c \in \mathbb{R}$ st $c \leq B_n \; \forall n \in \mathbb{N}$ then $c \leq b$
simularly, If $A_n \leq c \; \forall n \in \mathbb{N}$, then $a \leq c$

Every Convergent Sequence is Bounded - feb 4th notes

Archinmedian Property

Given any $x \in \mathbb{R}$, $\exists n \in \mathbb{N}$ st $n > x$ (no larget real number)
Given any $y > 0$, $\exists n \in \mathbb{N}$ st $0 < 1/n < y$ (no smallest positive number)

Examples of Proofs/other Questions

Sequence Convergence based of Def of Convergence

1) Let ε>0 be arbitray.
2) Choose N (depend on ε). solve for a n in terms of ε
3) Assume $n \geq N$, and show that the claim is true

examples: homework #3

Homework 2 - Inf and Sup based Proofs

look over and rework homework 2 problems.