Relativity 1

Disclaimer: These lecture notes are not meant to replace the course textbook. The content may be incomplete. Some topics may be unclear. These notes are only meant to be a study aid and a supplement to your own notes. Please report any inaccuracies to the professor.

Newtonian Relativity

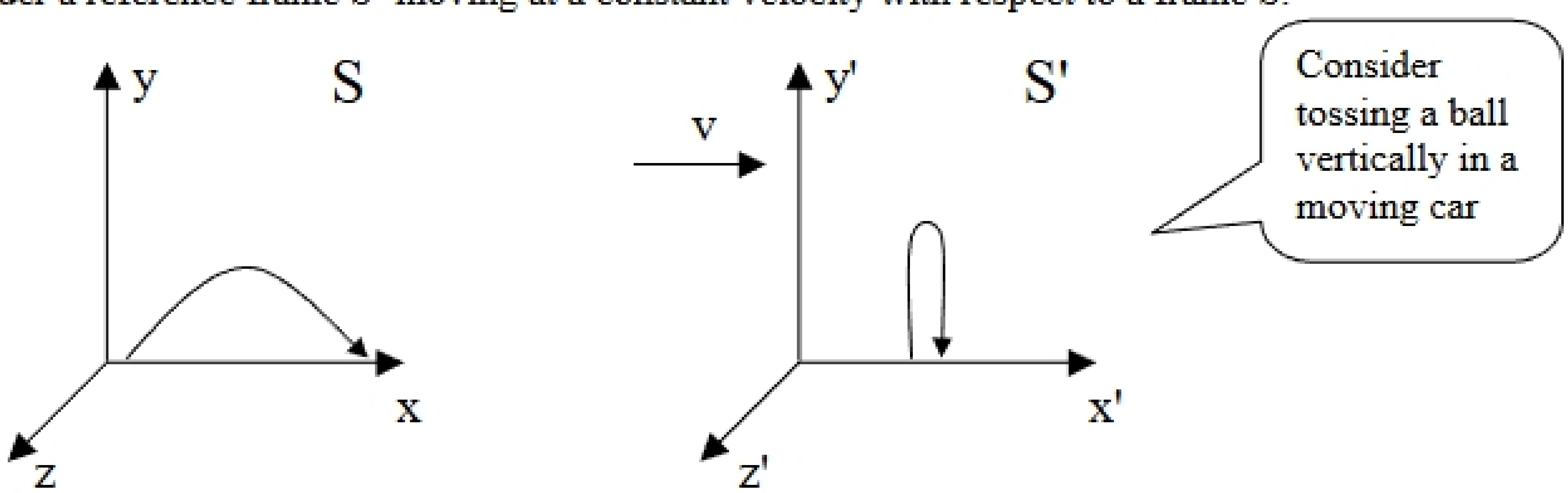
Galileo and Newton described the motion of objects with respect to a particular **reference frame**, which is basically a coordinate system attached to a particular observer.

A **reference frame** in which Newton's Laws hold is called an **inertial frame**. It is a frame that is not accelerating.

Newtonian Principle of Relativity (Galilean Invariance):
If Newton's Laws hold in one inertial frame, they also hold in a reference frame moving at a constant velocity relative to the first frame. So the other frame is also an inertial frame. We can see this if we make a Galilean transformation:

Galilean Transformation

Consider a reference frame S' moving at a constant velocity with respect to a frame S:



$$x = x' + vt' \qquad x' = x - vt$$
$$y = y' \qquad y' = y$$
$$z = z' \qquad z' = z$$
$$t = t' \qquad t' = t$$

These transformation equations show you how to convert a coordinate measured in one reference frame to the equivalent coordinate in the other reference frame. Implicit in a Galilean transformation is that time is universal (time runs at the same rate in all frames).

Now consider the action of a force in one reference frame. For example, the force of gravity causes a dropped ball to accelerate:

y component:

$$F_y' = ma_y' = m\frac{d^2y'}{dt'^2}$$

But since $y' = y$ (and $t' = t$)

$a_y' = a_y$ and $F_y' = F_y$

x component:

$$F_x' = ma_x' = m\frac{d^2x'}{dt'^2} = m\frac{d^2}{dt^2}(x - vt) = m\frac{d^2x}{dt^2} = F_x$$

$a_x' = a_x$ and $F_x' = F_x$

Since the acceleration of the ball is the same in each reference frame, and thus the force acting on the ball, Newton's Laws are valid in both frames. Each is an inertial frame.

Note that since the force is identical in each frame, there is no way to detect which frame is moving and which is not. You can only detect relative motion. For example, if a jet flies west at 1000 mph at the equator, is the jet moving or is the Earth moving? The jet flies over the surface of the Earth, but with respect to the Sun the jet is not moving and the Earth is turning beneath it! The fact that we cannot detect absolute motion is known as ***Relativity***. It is only relative motion that matters.

Example: Consider tossing a ball forward from a moving car at a velocity v' with respect to the reference frame of the car. What is the velocity of the ball with respect to the sidewalk along the road?

We need to know how to transform velocities. If we assume that the car is moving along the x axis at a velocity v with respect to the reference frame of the road, then

$$x = x' + vt$$

according to a Galilean Transformation. If we differentiate this with respect to time, we get:

$$\frac{dx}{dt} = \frac{dx'}{dt} + v$$

So, the velocity of the ball is $v'+v$. It is the sum of the car's velocity and the velocity of the ball with respect to the car. This should agree with our common sense.

Now suppose that instead of a ball we throw a light beam forward from the car. Light is an electromagnetic wave, and according to Maxwell's Equations it travels at a velocity $c = 3.0\times10^8\ \mathrm{m/s}$ in vacuum. For example, you could derive the following equation:

$$\frac{\partial^2 E_x}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\Rightarrow E_x = E_0 \sin[k(x - vt)] \quad \text{where} \quad v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

This is the velocity with respect to the car, but what about the velocity with respect to the sidewalk? Would it be $c+v$? That would agree with our common sense, but not with Maxwell's Equations. Maxwell's Equations state that the speed of light is c and only c, but for which reference frame does it refer to?

For clarification of the issue, let's consider **sound waves**, which are another form of a traveling wave. Sound waves are pressure waves. Pressure is a measure of how hard molecules push on a wall (force per unit area), so obviously you need some molecules around to have pressure, and thus pressure waves. Therefore, sound waves require a medium to propagate. The speed of sound at normal temperature and pressure in air is 343 m/s, or about 765 mph.

In our car example, we could consider honking the horn instead of turning on the headlights. We then create sound waves that propagate forward from the vehicle. The speed of sound is 343 m/s as we have noted, but this is the speed with respect to the propagation medium. If the air is still with respect to the sidewalk, then the speed of the sound wave is 343 m/s with respect to the sidewalk and **not** the horn on the car. In fact, if our car had a rocket strapped on, it could exceed the speed of sound and overtake its own emitted sound wave. (You then create a shock wave, which gives rise to a sonic boom. By the way, this car experiment was actually done recently!)

So honking the horn is not the same as tossing a ball forward. The velocity of the sound waves is always 343 m/s with respect to the reference frame of the air. It did not obey a Galilean Transformation because it picks out a special reference frame.

Now let's return to the light from the car's headlights. If the light beam acts like a ball thrown forward, we would measure a different velocity for the light depending on whether we were in the car or on the sidewalk. Also, Maxwell's Equations would have to be modified to account for a velocity different than c for all reference frames other than the one it apparently describes (does it apply to the car reference frame or the sidewalk?) If the light waves act like sound waves, then what is the propagation medium? Is it the air on the Earth? It can't be because light can propagate in a vacuum. So instead, it was proposed in the 19th century that light propagates through **ether**—some sort of medium—although nobody knew what this ether was. It was supposed that this ether might be at rest with respect to the solar system, or maybe the galaxy. In any case, the Earth would move through this ether, and we should observe light traveling at a speed different than c.

The issue was settled experimentally, but it took Einstein to put everything in the right perspective. The answer will surprise you. Light waves do act like balls thrown forward, but the assumptions behind the Galilean Transformation are wrong. Not only that, there is nothing wrong with Maxwell's Equations. The speed of light is always c no matter who measures it. But I'm getting ahead of myself…