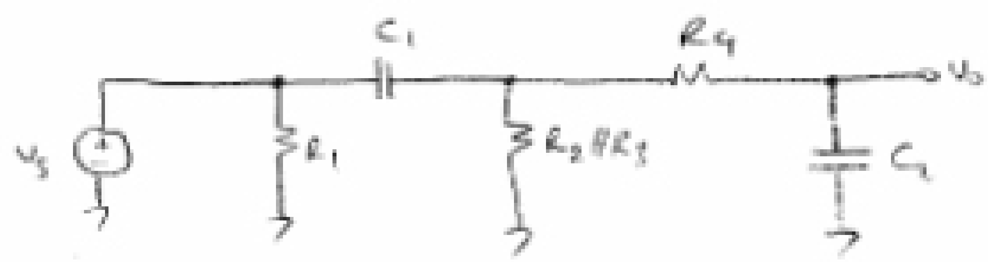
ELEN 325 HW#1 SOLUTION

1. (a)



$$\frac{V_o}{V_s}(s) = \frac{R_2 // R_3 // \left(R_4 + \frac{1}{sC_2}\right)}{\frac{1}{sC_1} + \left(R_2 // R_3 // \left(R_4 + \frac{1}{sC_2}\right)\right)} \cdot \frac{\frac{1}{sC_2}}{R_4 + \frac{1}{sC_2}}$$

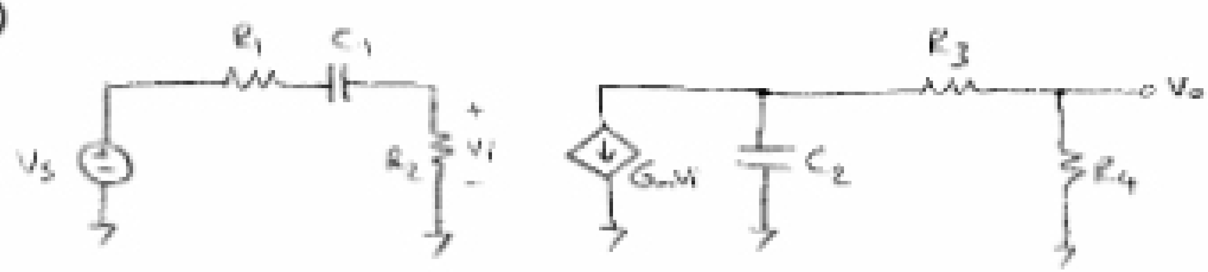
$$= \frac{\frac{(R_2//R_3)\left(R_4 + \frac{1}{sC_2}\right)}{(R_2//R_3) + \left(R_4 + \frac{1}{sC_2}\right)}}{\frac{1}{sC_1} + \frac{(R_2//R_3)\left(R_4 + \frac{1}{sC_2}\right)}{(R_2//R_3) + \left(R_4 + \frac{1}{sC_2}\right)}} \cdot \frac{\frac{1}{sC_2}}{R_4 + \frac{1}{sC_2}}$$

$$= \frac{(R_2//R_3)\left(R_4 + \frac{1}{sC_2}\right)}{\frac{1}{sC_1}\left((R_2//R_3) + R_4 + \frac{1}{sC_2}\right) + (R_2//R_3)\left(R_4 + \frac{1}{sC_2}\right)} \cdot \frac{\frac{1}{sC_2}}{R_4 + \frac{1}{sC_2}} \quad \begin{matrix} \times\, sC_1 \\ \times\, sC_1 \end{matrix}$$

$$= \frac{sC_1(R_2//R_3)}{sC_2(R_2//R_3) + sC_2R_4 + 1 + s^2C_1C_2R_4(R_2//R_3) + sC_1(R_2//R_3)}$$

$$= \frac{sC_1(R_2//R_3)}{s^2C_1C_2R_4(R_2//R_3) + s\left(C_1(R_2//R_3) + C_2(R_2//R_3) + C_2R_4\right) + 1}$$

(b)



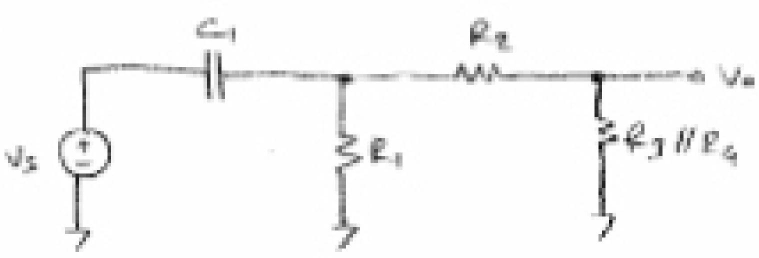
$$\frac{V_i}{V_s}(s) = \frac{R_2}{R_1 + \frac{1}{sC_1} + R_2} = \frac{s \cdot \frac{R_2}{R_1 + R_2}}{s + \frac{1}{C_1(R_1+R_2)}}$$

$$V_o = -G_mV_i\left(\frac{1}{sC_2} // (R_3 + R_4)\right) \cdot \frac{R_4}{R_3 + R_4}$$

$$\frac{V_o}{V_i}(s) = -G_m \frac{\frac{1}{sC_2}(R_3+R_4)}{\frac{1}{sC_2}+R_3+R_4} \cdot \frac{R_4}{R_3+R_4} = \frac{-G_m R_4}{sC_2(R_3+R_4)+1}$$

$$\frac{V_o}{V_s}(s) = \frac{V_i}{V_s}(s) \cdot \frac{V_o}{V_i}(s) = \frac{s}{s+\frac{1}{C_1(R_1+R_2)}} \cdot \frac{1}{1+sC_2(R_3+R_4)} \cdot \frac{-R_2 G_m R_4}{R_1+R_2}$$

(c)



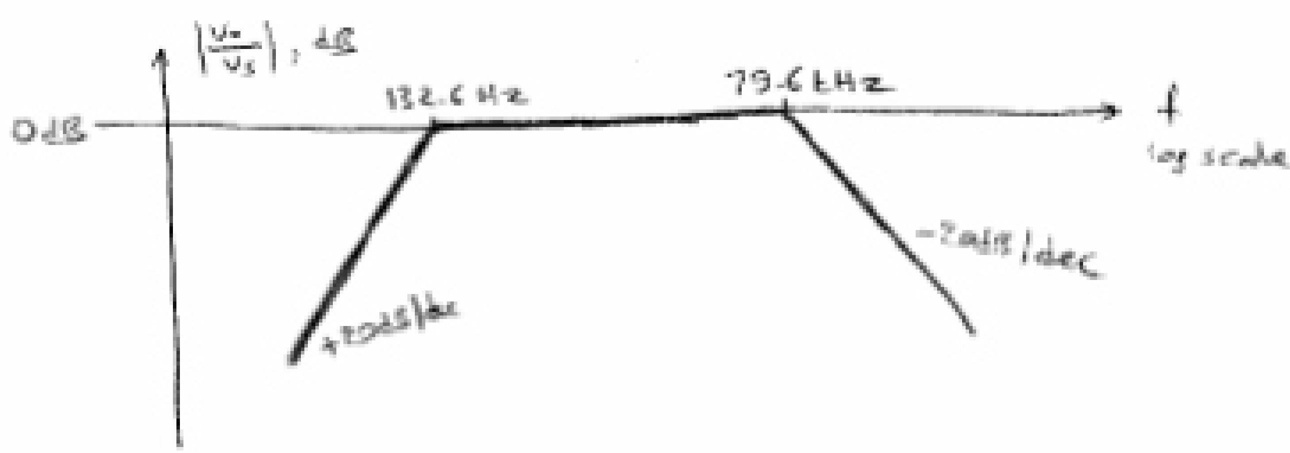
$$\frac{V_o}{V_s}(s) = \frac{R_1 // (R_2 + (R_3 // R_4))}{\frac{1}{sC_1} + (R_1 // (R_2 + (R_3 // R_4)))} \cdot \frac{R_3 // R_4}{R_2 + (R_3 // R_4)}$$

$$= \frac{s}{s + \frac{1}{C_1(R_1 // (R_2 + (R_3 // R_4)))}} \cdot \frac{R_3 // R_4}{R_2 + (R_3 // R_4)}$$

2. (a) $$\frac{V_o}{V_s}(s) = \frac{s\,(1\mu)(3k // 2k)}{s^2 (1\mu)(1n)(2k)(3k // 2k) + s\cdot\left((1\mu)(3k // 2k) + (1n)(3k // 2k) + (1n)(2k)\right) + 1}$$

$$= \frac{5\times10^5\, s}{s^2 + 5\times10^5\, s + 4.167\times10^8} = \frac{5\times10^5\, s}{(s+833)(s+5\times10^5)}$$

$$= \frac{s}{s + 2\pi\, 132.6} \cdot \frac{1}{1+\frac{s}{2\pi\, 79.6\times10^3}}$$



$$\left|\frac{V_o}{V_i}(j\omega)\right|_{\omega=2\pi 10^5} = \left|\frac{1}{1 - j\,\frac{2\pi\, 132.6}{2\pi\, 10^5}}\right| \cdot \left|\frac{1}{1 + j\,\frac{2\pi\, 10^5}{2\pi\, 79.6\times 10^3}}\right|$$

$$= \frac{1}{\sqrt{1+\left(\frac{132.6}{10^5}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{10^5}{79.6\times10^3}\right)^2}} = 0.62$$

$$\angle \frac{V_o}{V_i}\bigg|_{\omega=2\pi 10^5} = \tan^{-1}\frac{2\pi\,132.6}{2\pi\,10^5} - \tan^{-1}\frac{2\pi 10^5}{2\pi 79.6\times10^3} = -51.4^\circ = -0.9 \text{ rad.}$$

$$V_{o,ac} = 2\times 0.62 \sin(2\pi 10^5 t - 0.9) = 1.24 \sin(2\pi 10^5 t - 0.9)$$

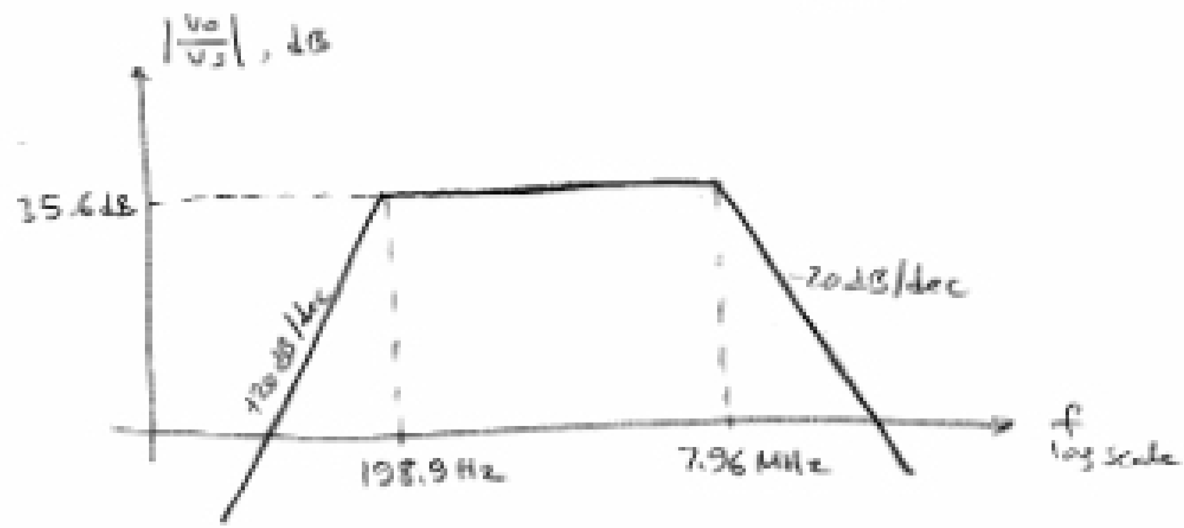
$$V_{o,dc} = 5 \cdot \frac{R_3}{R_2 + R_3} = 5 \cdot \frac{2k}{3k + 2k} = 2V$$

$$V_o(t) = 2 + 1.24 \sin(2\pi 10^5 t - 0.9)$$

(b)

$$\frac{V_o}{V_s}(s) = \frac{s}{s + \frac{1}{(100n)(2k+6k)}} \cdot \frac{1}{1 + s\cdot(10p)(1k \| 1k)} \cdot \frac{-(6k)(80m)(1k)}{(2k+6k)}$$

$$= \frac{s}{s + 2\pi\, 198.9} \cdot \frac{1}{1 + \frac{s}{2\pi\, 7.96\times10^6}} \cdot (-60)$$



20 log 60 = 35.6 dB

$$\left|\frac{V_o}{V_i}(j\omega)\right|_{\omega=2\pi 10^3} = \left|\frac{1}{1 - j\,\frac{2\pi\,198.9}{2\pi\,10^3}}\right| \cdot \left|\frac{1}{1 + j\,\frac{2\pi\,10^3}{2\pi\,7.96\times10^6}}\right| \cdot 60$$

$$= \frac{1}{\sqrt{1+\left(\frac{198.9}{1000}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{1}{7960}\right)^2}} \cdot 60 = 58.8$$