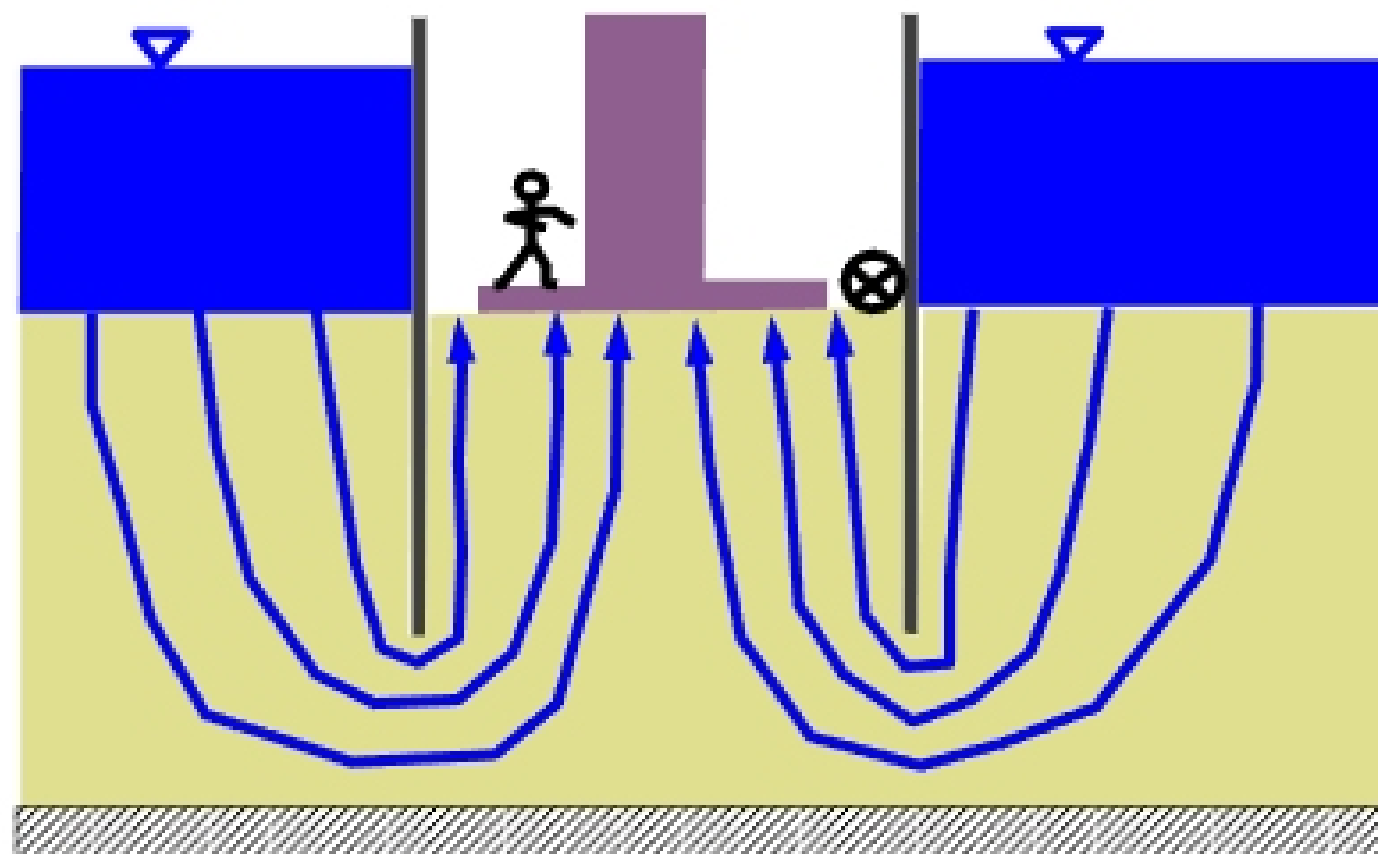


## Period #9: Multi-dimensional Fluid Flow in Soils

### A. Objective:

- To this point in time, we have considered *one-dimensional* flow in soils, where all fluid is flowing *in the same direction*.
- In most cases, however, fluid in different regions will be flowing in different directions. We call this *multi-dimensional flow*.
- Our objective here is to learn how to solve multi-dimensional flow problems such as the one shown below.



Multi-dimensional flow situation.

- To develop these capabilities, we will use two considerations:
  - 1) conservation of fluid mass in the soil (or *continuity*);
  - 2) *multi-dimensional* forms of Darcy's Law.

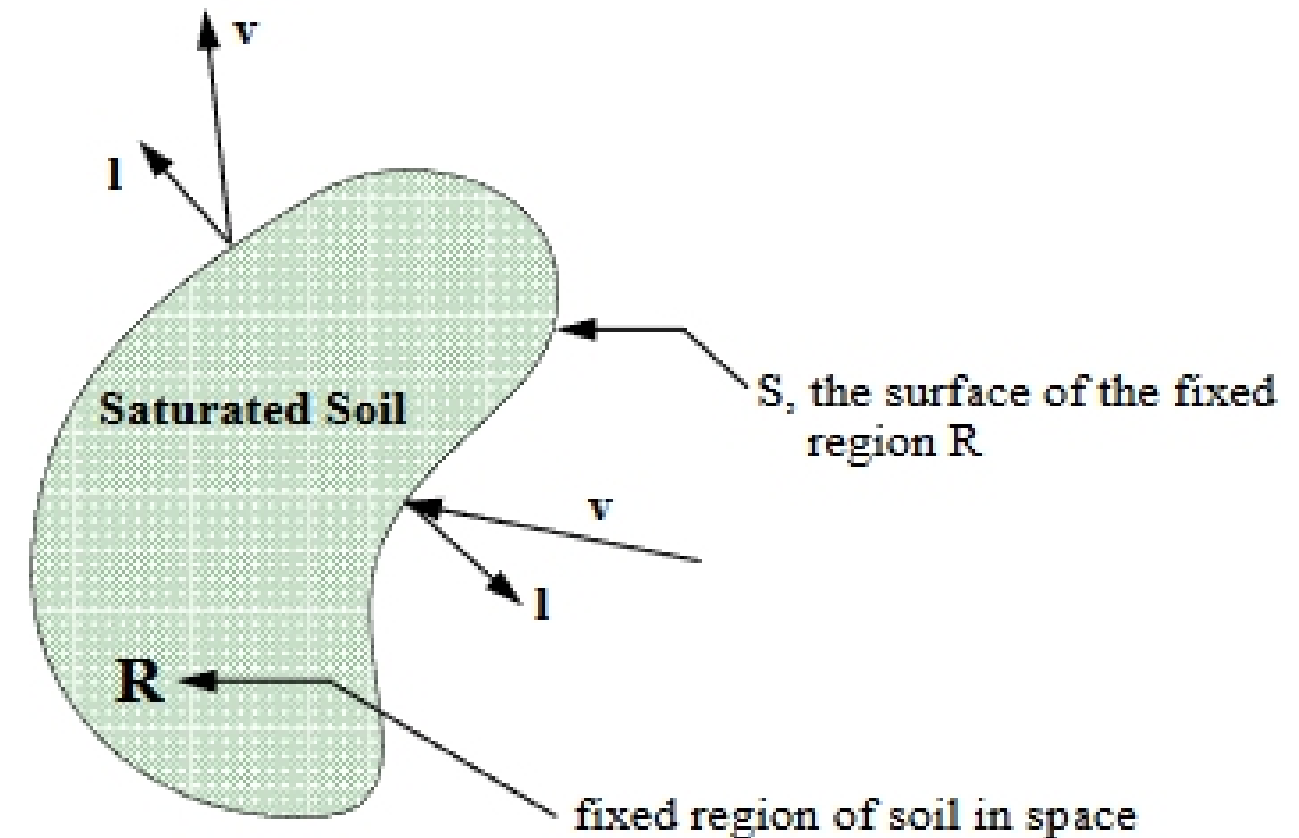
## B. Conservation of Fluid Mass (or *Continuity*)

Consider the case of fluid flowing through a region of soil in space as shown below:

Mass of water  $M_w$  in R:

$$M_w = \int_R n \rho_w dR, \text{ where:}$$

- $\rho_w$  is the mass density of water (constant)
- $n$  is the porosity of the soil (constant)



Net rate of fluid mass outflow from R:

$$\text{rate of mass outflow} = \int_S \mathbf{l} \cdot (\rho_w \mathbf{v}) dS, \text{ where:}$$

- $\mathbf{v}$  is the discharge velocity;
- $\mathbf{l}$  is the unit normal vector.

A statement for the conservation of fluid mass in  $R$  is the following:

net rate of mass outflow = rate of fluid mass decrease

$$\begin{aligned} \int_S \mathbf{l} \cdot (\rho_w \mathbf{v}) dS &= -d(M_w)/dt \\ &= -\int_R \partial(n\rho_w) / \partial t dR \\ &= 0, \text{ since } n \text{ and } \rho_w \text{ are both assumed to be constant.} \end{aligned}$$

Assumption: both the soil and the fluid  
do not compress during the flow.

$\therefore$  rate of fluid mass outflow from  $R = 0$

Restated,  $q_{in} = q_{out}$

By Green's Theorem from Vector Calculus:

$$0 = \int_S \mathbf{l} \cdot (\rho_w \mathbf{v}) dS = \int_R \nabla \cdot (\rho_w \mathbf{v}) dR = \int_R [\mathbf{v} \cdot (\nabla \rho_w) + \rho_w \nabla \cdot \mathbf{v}] dR$$

Side note: The quantity  $\nabla$  denotes a gradient operator. For example,

$$\nabla \rho_w = \left[ \frac{\partial \rho_w}{\partial x_1} \right] \mathbf{e}_1 + \left[ \frac{\partial \rho_w}{\partial x_2} \right] \mathbf{e}_2 + \left[ \frac{\partial \rho_w}{\partial x_3} \right] \mathbf{e}_3.$$

(Since  $\rho_w$  is assumed constant and uniform,  $\nabla \rho_w = \mathbf{0}$ .)