Laboratory Experiment #1

Measurement of Kinematic Viscosity

Marian Muste, Fred Stern

1. Purpose

To measure the kinematic viscosity of a fluid, the uncertainty of the measurement, to compare the measured kinematic viscosity with manufacturer's value, and to demonstrate the effects of viscosity by comparison of the fall times for spheres of different densities and diameters.

2. Experiment Design

A fluid deforms continuously under the action of a shear stress (http://css.engineering.uiowa.edu/fluidslab/referenc/concepts.html - select Viscosity). The rate of strain in a fluid is proportional to the shear stress. The proportionality constant is the *dynamic viscosity* (μ). Viscosity is a thermodynamic property that varies with pressure, temperature, and fluid nature. For instance, for a given state of pressure and temperature, there is a variation of three orders of magnitude between water and glycerin, the fluid which will be used in this experiment. The *kinematic viscosity* $\nu = \mu/\rho$ where ρ is the density of the fluid, is most frequently used in the equations of fluid mechanics.

Common methods used to determine viscosity are the rotating-concentric-cylinder method (Engler viscosimeter) and the capillary-flow method (Saybolt viscosimeter). Alternatively, we will measure the kinematic viscosity through its effect on a falling object. The maximum velocity attained by an object in free fall (terminal velocity) is strongly affected by the viscosity of the fluid through which it is falling. When terminal velocity is attained, the body experiences no acceleration, so the forces acting on the body are in equilibrium (Figure 1).

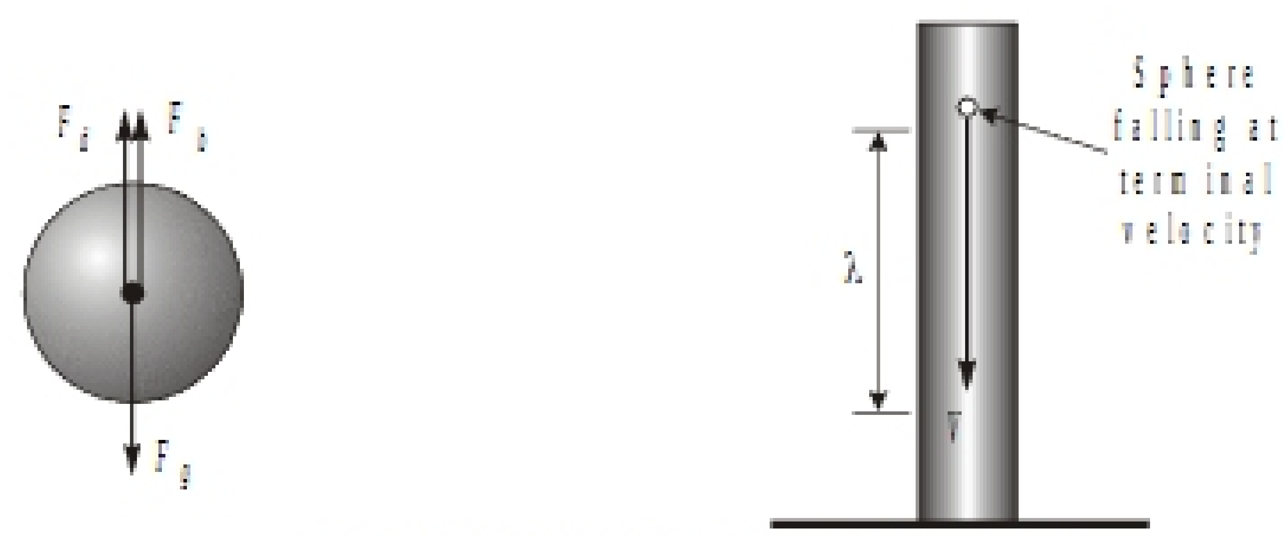

Figure 1. Experimental arrangement

The forces acting on the body are the gravitational force,

$$F_g = mg = \rho_{sphere}\,\pi\,\frac{D^3}{6}\,g \tag{1}$$

The force due to buoyancy,

$$F_b = \rho_{fluid}\,\pi\,\frac{D^3}{6}\,g \tag{2}$$

and the resistance of the fluid to the motion of the body, similar to friction. For $Re = VD/\nu << 1$ (Re is a dimensionless number that characterizes the flow), the drag force on a sphere is described by the Stokes expression

$$F_d = 3\,\rho_{fluid}\,\pi\,\nu\,V\,D \tag{3}$$

where D is the sphere diameter, ρ_{fluid} is the density of the fluid, ρ_{sphere} is the density of the falling sphere, ν is the viscosity of the fluid, F_d, F_b, and F_g, denote the drag, buoyancy, and weight forces, respectively, V is the velocity of the sphere through the fluid (in this case, the terminal velocity), and g is the acceleration due to gravity (White 1994).

Once terminal velocity is achieved, a summation of the vertical forces must balance. Equating the forces gives:

$$\nu = \frac{D^2 g(\rho_{sphere}/\rho_{fluid} - 1)t}{18\lambda} \quad (4)$$

where t is the time for the sphere to fall the vertical distance l .

Using equation (4) for two different balls, namely, teflon and steel spheres, the following relationship for the density of the fluid is obtained, where subscripts s and t refer to the steel and teflon balls, respectively.

$$\rho_{fluid} = \frac{D_t^2 t_t \rho_t - D_s^2 t_s \rho_s}{D_t^2 t_t - D_s^2 t_s} \quad (5)$$

3. Experiment Process

3.1 Setup

The equipment (measurement systems) in the experiment includes:

- A transparent cylinder (beaker) containing glycerin. A scale is attached to its side to read the distance the sphere has fallen.
- Teflon and steel spherical balls of different sizes
- Stopwatch
- Micrometer
- Thermometer

3.2 Data acquisition

In this experiment, we will allow a sphere to fall through a long transparent cylinder filled with the glycerin Figure 1. After the sphere has fallen a long enough distance so that it achieves terminal velocity, we will measure the length of time required for the sphere to fall through the distance l .

The experiment procedure follows the sequence described below:

1. Measure the temperature of the room.
2. Two horizontal lines are marked on the vertical cylinder. Measure the distance between the two lines, l.
3. Measure the diameter of each sphere (teflon and steel) using the micrometer.
4. Release the sphere at the surface of the fluid in the cylinder. Then, release the gate handle.
5. Release the spheres, one by one
6. Measure the time for the sphere to travel the length l
7. Repeat steps 3- 6 for all spheres. At least 10 measurements will be taken for each type of spheres.

Since the fall time of the sphere is very short, it is important to measure the time as accurately as possible. Start the stopwatch as soon as the bottom of the ball hits the first mark on the cylinder and stop it as soon as the bottom of the ball hits the second mark. Two people should cooperate in this measurement with one looking at the first mark and handling the stopwatch, and the other looking at the second mark. A spreadsheet will be provided to record the measurements.

3.3 Data reduction

Figure 2 illustrates the block diagram of the measurement systems and data reduction equations for the results. A spreadsheet will be provided to conduct the data reduction.

Data reduction includes the following steps

1. Calculate the flu... ng equation (5).
2. Calculate the ki... ent using equation (4) for either sphere type.

3.4 Uncertainty assessment

Uncertainties f... cosity will be evaluated.

The methodolo... Standard (AIAA, 1995) as summarized in Stern et al. (1999) for multiple tests (M = 10). The block diagram for propagation of errors in the measured density and v... or density and viscosity of glycerin are equation (5... ndependent variable, Xi, in data reduction equation... bias errors) and repeated measurements (for prec... rs assumed for the present experiment.

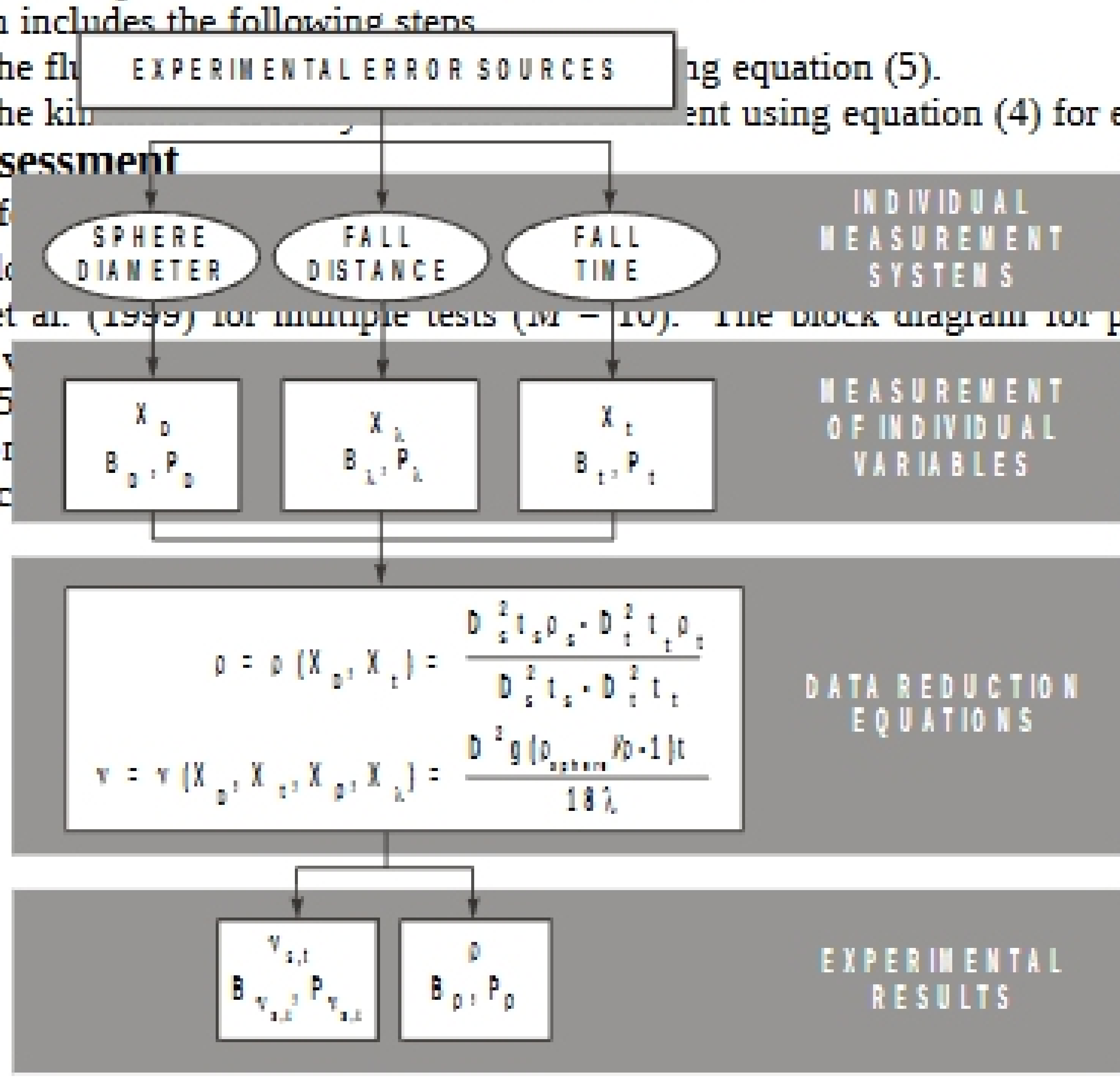

Figure 2. Block diagram of density/viscosity experiment including: measurement systems, data reduction equations, and results

Table 1. Assessment of the bias limits for the independent variables

Bias limit	Bias Limit	Estimation
$B_D = B_{Ds} = B_{Dt}$	0.000005 m	½ instrument resolution
$B_t = B_{ts} = B_{tt}$	0.01 s	Last significant digit
$B_{\square}$	0.00079 m	½ instrument resolution

The bias limit, precision limit, and overall uncertainty for the experimental results, namely the density and viscosity of glycerin, are then found using Eqs. (14), (23) and (24) in Stern et al. (1999). Note that in the present analysis we will neglect the contribution of the correlated bias errors in equation (14).

<u>Density of glycerin</u>

The total uncertainty for the density measurement is:

$$U_{\bar{\rho}_G} = \sqrt{B_{\rho_G}^2 + P_{\bar{\rho}_G}^2} \qquad (6)$$

The bias limit B_{ρ_G}, and the precision limit $P_{\bar{\rho}_G}$, for the result are given by:

$$B_{\rho_G}^2 = \sum_{i=1}^{j} \theta_i^2 B_i^2 = \theta_{D_t}^2 B_{D_t}^2 + \theta_{t_t}^2 B_{t_t}^2 + \theta_{D_s}^2 B_{D_s}^2 + \theta_{t_s}^2 B_{t_s}^2 \qquad (7)$$

$$P_{\bar{\rho}_G} = \frac{2 \cdot S_{\bar{\rho}_G}}{\sqrt{M}} \qquad (8)$$

where the sensitivity coefficients (calculated using mean values for the independent variables) are:

$$q_{D_t} = \frac{\partial \rho_G}{\partial D_t} = \frac{2 D_s^2 t_t t_s D_t (\rho_s - \rho_t)}{\left(D_t^2 t_t - D_s^2 t_s\right)^2} \left[\frac{kg}{m^4}\right] \qquad q_{t_t} = \frac{\partial \rho_G}{\partial t_t} = \frac{D_s^2 D_t^2 t_s (\rho_s - \rho_t)}{\left(D_t^2 t_t - D_s^2 t_s\right)^2} \left[\frac{kg}{m^3 s}\right]$$

$$q_{D_s} = \frac{\partial \rho_G}{\partial D_s} = \frac{2 D_t^2 t_t t_s D_s (\rho_t - \rho_s)}{\left(D_t^2 t_t - D_s^2 t_s\right)^2} \left[\frac{kg}{m^4}\right] \qquad q_{ts} = \frac{\partial \rho_G}{\partial t_s} = \frac{D_s^2 D_t^2 t_t (\rho_t - \rho_s)}{\left(D_t^2 t_t - D_s^2 t_s\right)^2} \left[\frac{kg}{m^3 s}\right]$$