Centripetal Force

1. Introduction

When an object travels in a circle, even at constant speed, it is undergoing acceleration. In this case the acceleration acts not to increase or decrease the *magnitude* of the velocity vector, but rather to change its *direction*. Newton's second Law tells us that in the absence of any outside Force, an object will travel at constant speed in a *straight* line. Therefore if we observe the direction of motion changing, we know there is a Force acting. The direction of that Force is the direction that the velocity vectors turns toward. In the case of circular motion, we can see that the acceleration in inward since the direction of motion is forever turning inward toward the center.

We can show that in order for an object to continue moving in a circle that is at a constant radius, at constant speed, the Force must exactly match the mass, speed and radius of the circle according to the equation

$$F = m\frac{v^2}{R}$$

As shown in Figure 1, if the Force is too small or the object traveling too fast, then it will move outward from the circular path. If the object travels too slowly or the Force is too large, then it will fall inward toward the center. For a given speed and radius, there is only one magnitude of Force which is exactly right for keeping the object moving in a circle. From the geometry of the situation, we could prove that this magnitude is mv^2/R.

There are many cases of circular motion: orbits of planets and satellites are nearly circular, we can talk about a person on a merry-go-round or a child playing on a rotating platform, we might calculate the motion of electrons or protons in a particle accelerator. The Forces acting in these cases are gravitational forces, friction or electromagnetic forces. Whatever their origin, because they act to keep objects moving in a circle, we shall call them the *Centripetal Force*, F_c and call the acceleration they produce *Centripetal Acceleration*, a_c.

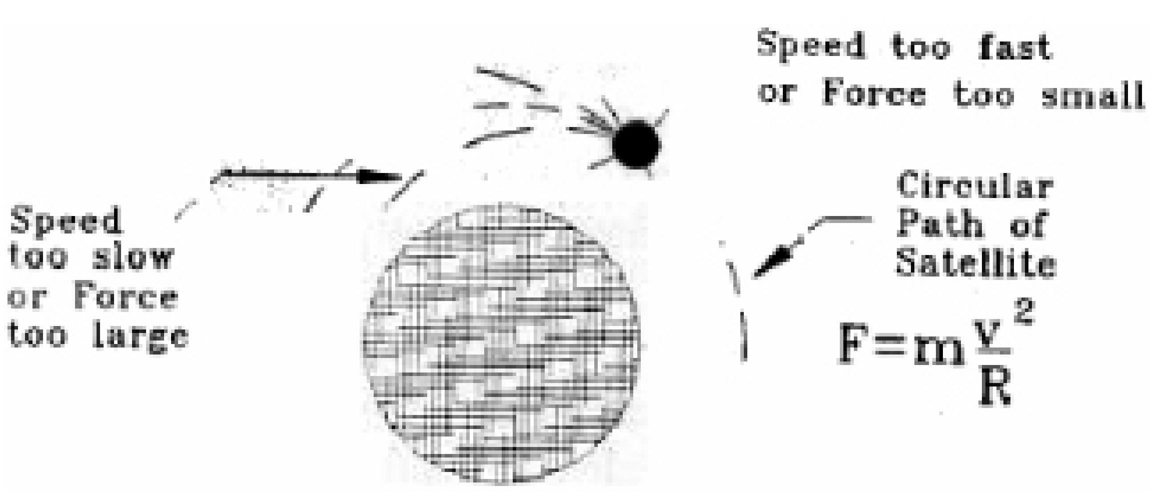

Figure 1: A satellite orbiting the Earth must travel at exactly the right speed and height so that the gravitational Force provides the desired Centripetal acceleration.

The word Centripetal comes from Latin and means "center seeking" because the Forces and accelerations must point exactly to the center of the circle in order for there to be circular motion. *Because we see that these Forces and accelerations produce circular motion, we know their magnitudes must be:*

$$F_c = m\frac{v^2}{R}$$

$$a_c = \frac{v^2}{R}$$

We can also relate the centripetal acceleration and Force to the period of rotation, T, since

$$v = \frac{2\pi R}{T}$$

it follows that

$$F_c = ma_c = m\frac{v^2}{R} = m\frac{(2\pi R)^2}{RT^2} = m\frac{4\pi^2 R}{T^2}$$

OUR EXPERIMENT

In this experiment we will measure the force required to keep a mass moving at constant angular velocity in a circle of constant radius. This force will be applied by a spring suspended between the mass and a post at the center of the circle about which the mass rotates. The general outline of the apparatus is shown in Figures 2 and 3. A Bob is suspended from a crossarm such that with no other masses or springs attached to it, it hangs straight down over a Pointer. When the system is at rest and we attach a spring to the Bob, a hanging mass is needed to keep the spring stretched and the Bob directly over the pointer. If we remove this mass, then the spring will pull the Bob inward towards the axis. However if we begin to spin the Bob about the axis, its inertia will try to make it go in a straight line and the Spring will have to pull inward to keep it moving in a circle. Thus *the spring provides the Centripetal Force*. By **Newton's Third Law**, the Bob applies an equal and opposite Force to the spring, and so the Spring will begin to stretch outward. As the rotational speed increases, the spring must apply more and more Force according to the equation $F = mv^2/R$. The spring therefore stretches until at exactly the right rotational velocity the Bob once again hangs over the Pointer (Figure 4.6).

For this experiment we must rotate the apparatus at constant speed while trying to measure the period of rotation. This is difficult to achieve and often the experimental results have significant uncertainties for this experiment. Try as best you can to measure the frequency of rotation for a constant speed by keeping the bob rotating at a constant radius.

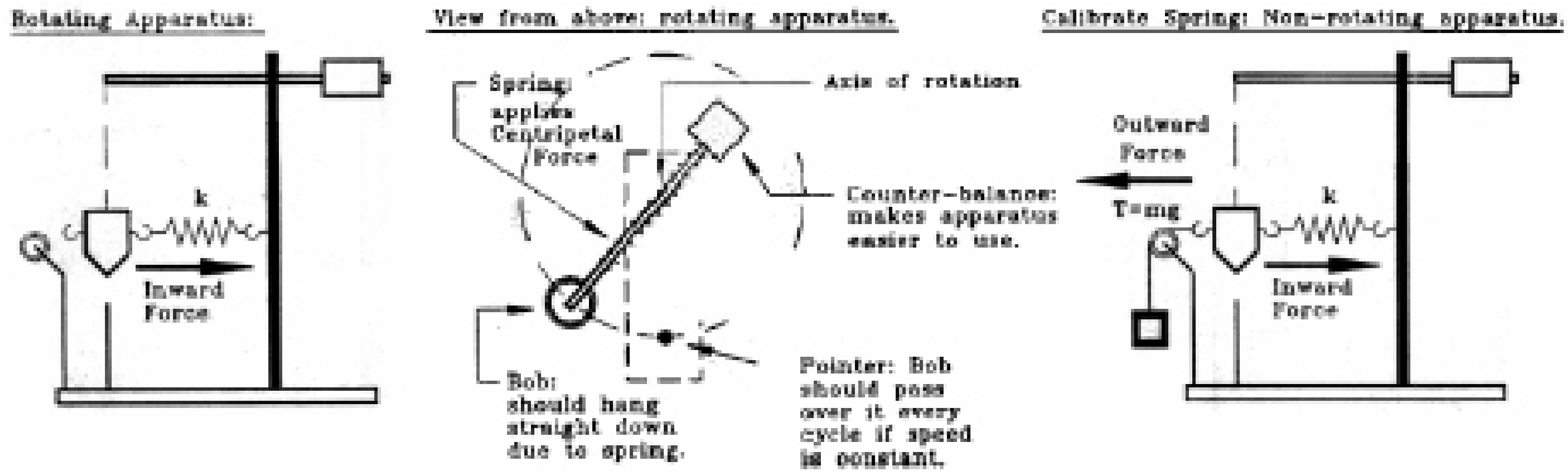

Figure 2: The Centripetal Force Apparatus.

2 Procedure

OPERATION OF THE CENTRIPETAL FORCE APPARATUS

The rotor or axis of the apparatus is rolled by hand between your thumb and forefingers. When the rotational speed is just right, the bob hanging from the cross-arm will hang straight down over the pointer, the Centripetal Force required for uniform circular motion is then supplied by the spring. The purpose of the counterweight is to balance the bob so that the apparatus spins smoothly. When properly balanced, it applies no force.

INITIAL SETUP:

• Remove the bob, weigh it, and record its mass. If additional masses are available to add to the bob, now is the time to weigh them as well. Weigh the mass holder hook and any masses you will use with it as well. Remember to record an estimated uncertainty for these values according to how well you think you can read the scales.

Bob Mass:________________________
Mass Holder Hook :____________________

CALIBRATION OF THE SPRING

If the spring is an "'ideal" spring, the force it can apply is equal to $F = -k \times \Delta R$, where k is a constant known as the "Spring Constant" or "Elastic Constant" and ΔR is the distance the spring stretches or compresses from its relaxed, equilibrium length. In other words, as you pull on or compress the spring, the force with which it "pushes back" increases linearly. This is known as "Hook's Law", but it is not an actual Law of Physics, but merely a model for the spring which holds fairly well as long as the spring is not compressed or stretched too much or too little.

• Set up the apparatus as shown in Figure 1. Remove the mass holder hook. Move the cross-arm so that the black bob is hanging straight down. Line up the vertical pointer directly under the tip of the bob as a reference for your measurement. Measure the distance from the pointer to the center of the spindle as accurately as you can. This is the equilibrium distance (the unstretched length) of the spring. Record this length as "R_0". This distance R_0 corresponds to a force, F_{spring}, of zero being applied to the spring.

R_0:_____________________