

Chapter 4 – Nominal and Effective Interest Rates

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- Nominal and Effective Interest Rates
- Comparing Payment Period and Compounding Period Lengths (PP vs. CP)
- Interest Periods Equal to Payment Periods
- Interest Periods Smaller than Payment Periods
- Interest Periods Larger than Payment Periods
- Interest Rates That Vary over Time

Nominal and Effective Interest Rates

- Generally interest can be quoted in more than one way:
 1. Quotation using a Nominal Interest Rate (or APR - the Annual Percentage Rate)
 2. Quoting an Effective Interest Rate (or APY – the Annual Percentage Yield)

Nominal and Effective Interest Rates

- Interest may be computed (compounded):
 - Annually – One time a year (at the end)
 - Every 6 months – 2 times a year (semi-annual)
 - Every quarter – 4 times a year (quarterly)
 - Every Month – 12 times a year (monthly)
 - Every Day – 365 times a year (daily)
 - ...
 - Continuous – infinite number of compounding periods in a year.

Terminology

Payment Period, T_p - Length of time during which cash flows are not recognized except as end of period cash flows.

Compounding Period (CP or interest period), T_c - Length of time between compounding operations.

Interest Rate Period, T - Interest rates are stated as % per time period. T is the time period.

Compounding frequency, m - the number of times that compounding occurs within the time period T .

Nominal Interest Rate

Nominal Interest Rate (r) - rate at which money grows, $\%/T$, without considering compounding.

It can be stated for any time period: 1 year, 6 months, quarter, month, week, day, etc.

To find equivalent nominal rate r for any other time period:

$$r = \text{interest rate per period} * \text{number of periods}$$

Nominal Interest Rate

For example, if the **nominal** interest rate, r , were:
 $r = 1.5\%$ per month,

the **nominal** rate would also be:

$r = 0.05\%$ per day, if 30 days/mo.

$r = 4.5\%$ per quarter

$r = 9\%$ semiannually

$r = 18\%$ per year

Effective Interest Rate

Effective Interest Rate (i) - rate at which money grows, $\%/T$, considering compounding.

- It is usually expressed on an annual basis (i_a), but any time basis can be used.
- Effective Interest Rate, i , equals the nominal interest rate, r , if the interest rate period T and the compounding period CP are equal.

- All the interest formulas, factors, tabulated values, and spreadsheet relations must have **the effective interest rate** to properly account for the time value of money (effect of compounding).
- Effective rate has the **compounding frequency** attached to the nominal rate statement.

Nominal Rates:

- Format: " r % per time period T "
- Ex: 5% per 6-months ($r = 5\%$, $T = 6$ months)

Effective Interest Rates:

- Format: " r % per time period T , compounded ' m ' times in time period T ."
- ' m ' denotes the number of times per T that interest is compounded.
- Ex: 18% per year, compounded monthly ($r = 18\%$, $T = 1$ year, $CP = 1$ month, $m = 12$)

$$\text{Effective rate per CP} = \frac{r\% \text{ per } T}{m \text{ per } T}$$

Example C4.1:

What is the CP , m , and effective rate per CP if

- a) $r = 9\%$ per year, compounded quarterly
- b) $r = 15\%$ per month, compounded daily

Solution

	<u>CP</u>	<u>m</u>	<u>Effective rate per CP</u>
a)	quarter	4	$9\%/4 = 2.25\%$
b)	day	30	$15\%/30 = 0.5\%$

Example C4.2:

Given $r = 9\%$ per year compounded monthly, find the Effective Monthly Rate (effective rate per CP).

Solution:

$m = 12$ compounding periods within a year (T).

Effective Monthly Rate (effective rate per CP):
 $0.09/12 = 0.0075 = 0.75\%/month$

Example C4.3:

Statement: 4.5% per 6 months – compounded weekly.
Find the effective weekly rate (effective rate per CP).

Solution:

Nominal Rate $r = 4.5\%$.

$T = 6$ months.

$m = 26$ weeks per 6 months T

effective weekly rate is:

$0.045/26 = 0.00173 = 0.173\%$ per week

Varying Statements of Interest Rates

- Effective Rate is directly stated
 - Effective rate is 8.243% per year, compounded quarterly;
 - No nominal rate given;
 - Compounding frequency $m = 4$;
 - No need to calculate the true effective rate!
 - It is already given: 8.243% per year!

Varying Statements of Interest Rates

- Only the interest rate is stated
 - 8% per year.

No information on the frequency of compounding;
Must assume it is for one year!

Assume that "8% per year" is a true, effective annual rate!

Effective Interest Rate, i , equals the nominal interest rate, r , if the interest rate period T and the compounding period CP are equal.

Effective rate per payment period = $(1 + r/m)^m - 1$

r = nominal interest rate per payment period
 m = number of compounding periods per payment period

Example C4.4:

What is the future worth of \$100 after 1 year if a bank pays 12% interest per year compounded semiannually?

Solution: First find effective interest rate per year
 $r = 12\%$ per year (nominal); $T = 1$ year; CP = 6 months;
 $m = 2$; payment period PP = 1 year

Effective rate per payment period = $(1 + r/m)^m - 1$
 i per year = $(1 + 0.12/2)^2 - 1 = 0.1236 = 12.36\%$

$F = \$100 (1 + 0.1236) = \112.36