

Paper #4

Due 12/12/00

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Suppose that in some closed environment there are two similar species. Let the population of the first species be denoted by $x(t)$ and the second by $y(t)$. The logistic model for the population of these two species is then

$$\begin{aligned}x'(t) &= x(\varepsilon_1 - \sigma_1 x) \\y'(t) &= y(\varepsilon_2 - \sigma_2 y)\end{aligned}$$

where $\varepsilon_1, \varepsilon_2, \sigma_1,$ and σ_2 are positive constants. Additionally, let us suppose that these two species compete for the same food resource. In this case, the population of one species decreases the amount of food available to the second species, and hence slows its rate of growth. We can model this by modifying our equations to have the following form:

$$\begin{aligned}x'(t) &= x(\varepsilon_1 - \sigma_1 x - \alpha_1 y) \\y'(t) &= y(\varepsilon_2 - \sigma_2 y - \alpha_2 x)\end{aligned}$$

where α_1 and α_2 are also positive constants. In this paper, we shall examine the solutions of this system.

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1. Consider the following system:

$$\begin{aligned}x'(t) &= x(1 - x - y) \\y'(t) &= y(0.75 - y - 0.25x)\end{aligned}$$

- Show that this system has four equilibrium points. Find them.
- Determine the type and stability of each of the equilibrium points.
- Use the information in b. to draw the phase portrait of the system.
- Describe the long term behavior of the population in this system.

2. Consider the following system:

$$\begin{aligned}x'(t) &= x(1 - x - y) \\y'(t) &= y(0.5 - 0.25y - 0.75x)\end{aligned}$$

- Show that this system has four equilibrium points. Find them.
- Determine the type and stability of each of the equilibrium points.
- Use the information in b. to draw the phase portrait of the system.
- Describe the long term behavior of the population in this system.

3. (BONUS QUESTION) Consider the general system

$$\begin{aligned}x'(t) &= x(\varepsilon_1 - \sigma_1 x - \alpha_1 y) \\y'(t) &= y(\varepsilon_2 - \sigma_2 y - \alpha_2 x)\end{aligned}$$

- Show that this system has four equilibrium points. Find them.
- Show that there is exactly one equilibrium point for which both x and y are positive.
- Determine the type and stability of this equilibrium point in the case $\sigma_1 \sigma_2 - \alpha_1 \alpha_2 > 0$. Use this information to sketch the phase portrait of the system, and then to describe the long term behavior of the system.
- Determine the type and stability of this equilibrium point in the case $\sigma_1 \sigma_2 - \alpha_1 \alpha_2 < 0$. Use this information to sketch the phase portrait of the system, and then to describe the long term behavior of the system.

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Rules:

1. The assignment is due at the beginning of class on Tuesday, December 12, 2000.
2. This project is to be completed by teams of 2–4 students, and all students shall make a reasonable contribution to the solution of the problem. Separate from the assignment, each student shall hand in a sheet that describes the work of the group. This will be used in grading; students in the same group may receive different grades.
3. You may not discuss this project with other members of class, or with anyone outside of class. You may (only) discuss the project with the instructor.
4. Aside from the restrictions in (3) above, the use of outside references is acceptable, and indeed encouraged. However, all outside references need to be properly acknowledged.
5. Answers should be a well-written paper that describes the problem and the solution. All of the usual rules of English grammar and composition apply.
6. Papers need to be neat, clean, and paper-clipped or stapled. They do not need to be typed or written in ink, but they must be legible and easily readable.
7. Copying the work of another student or portions of a published work constitutes plagiarism. Plagiarism or any other form of academic dishonesty will be cause for immediate failure of the course.