

Test Total

Name _____

Test 4 Introduction to Discrete Mathematics 3450:208 Dr. Norfolk
July 11, 2008

Show your work.

1. **Tim's Treadmills** started manufacturing in the year 2000. Each of their products carries a 10-character code, as follows:

- The first 2 characters are *decimal digits*, representing the *month* of manufacture
- The second 2 characters are also *decimal digits*, representing the *year of manufacture* (from 2000 onwards)
- The last 6 characters are either *decimal digits*, or *upper-case letters*, except that the letter 'O' is not used.

(a) How many codes are possible?

5 points

(b) How many of the codes have 'A1' as the 5th and 6th characters, or 'B2' as the 7th and 8th characters?

5 points

(c) If Tim modifies the rules so that the first 5 of the last 6 characters must be different, and the last character matches one of the previous 5, how many are possible?

5 points

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2. Which is more likely: Getting *exactly* one head in 3 tosses of a fair coin, or getting a high-value card (T,J,Q,K or Ace), when drawing one card from a shuffled deck?

Explain your answer carefully.

10 points

3. Suppose that we have a class of 23 girls and 18 boys, and we wish to pick a project team of 10 children.

(a) How many teams are possible?

5 points

(b) How many teams consist of exactly 4 boys and 6 girls?

5 points

(c) If Graham and Kent must be on a team together, or not at all, how many teams are possible?

5 points

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4. For a positive integer n , and a real number x , define a function $f : A \rightarrow \mathbb{R}$ via $f(n, x) = n^x$.
Using appropriate set notation, write the *domain* A of f .

5 points

5. Define $g : \mathbb{R} \rightarrow \mathbb{R}$ via $g(x) = \begin{cases} x - \lfloor x \rfloor & \text{if } x < 0 \\ \lceil x \rceil - x & \text{if } 0 \leq x \end{cases}$ and $h : \mathbb{Z} \rightarrow \mathbb{R}$ via $h(n) = n + \frac{1}{2}$

- (a) Evaluate $g(-1.4)$ and $g(2.4)$.

5 points

- (b) Explain whether or not g is one-to-one.

5 points

- (c) Find and simplify an expression for $g \circ h$.

5 points

6. How many people must we interview to *guarantee* that at least one group of 12 people share the same birthday?

5 points

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