

Multiplying and Dividing Fractions

In this section we discuss the operations of multiplication and division in the rational numbers.

Multiplication. Given two fractions $\frac{a}{b}$ and $\frac{c}{d}$ we know that the product is computed by:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

That is, we just multiply the numerators and the denominators. Let us investigate why this simple formula works. We can do this with using the **area model**. Recall that to multiply whole numbers m and n in the area model, we compute the area of the rectangle with side lengths m and n . To multiply the proper fractions $\frac{a}{b}$ and $\frac{c}{d}$:

1. Draw a square with side length 1.
2. Cut the square into b rows and d columns.
3. Shade in the rectangle which has a rows for its height and c columns for its length.
4. Count the number of all the subrectangles ($b \cdot d$).
5. Count the number of rectangles in the shaded rectangle ($a \cdot c$).

See the diagrams on page 376. Similar procedures will show us how to model the multiplication of improper fractions.

Division. Given two fraction $\frac{a}{b}$ and $\frac{c}{d} \neq 0$ we can compute:

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$$

There are several ways to understand the division of fractions. One way is to use the missing factor model. To solve the problem $\frac{a}{b} \div \frac{c}{d} = ?$ we can instead ask

$$\frac{a}{b} = \frac{c}{d} \cdot (?).$$

We can see that the answer to this new problem is

$$(?) = \frac{d a}{c b}$$

since

$$\frac{c}{d} \cdot \frac{d a}{c b} = \frac{c d a}{d c b} = 1 \frac{a}{b} = \frac{a}{b}.$$

This establishes the formula.

We recall that in our discussion of division of whole numbers, we often viewed division in terms of grouping. A similar idea will work for fractions. (See example 6.11 on page 380).

Another option is to think of division as a partitioning. (See example 6.12 on page 381).