

Adding and Subtracting Fractions

Now that we have carefully defined the set of rational numbers we will describe how to do basic arithmetic in this set.

Addition.

Definition. For the fractions $\frac{a}{b}$ and $\frac{c}{b}$ the sum is given by

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}.$$

In words, if two fractions have the same denominator we may add them by adding the numerators.

It is helpful to illustrate this definition with some of the models of the previous section.

See page 364.

Since we now know how to add fractions with a common denominator we can add fractions with unlike denominators by doing the following procedure:

Procedure (Adding Fractions Whose Denominators are Different).

1. Rewrite both fractions over a common denominator
2. Add the numerators of the converted fractions and put the result over the common denominator
3. Reduce to lowest terms

In general we have:

$$\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d}{b \cdot d} + \frac{b \cdot c}{b \cdot d} = \frac{ad + bc}{bd}.$$

Fraction strips are an excellent way to demonstrate that this procedure works. See page 364.

Subtraction.

If two fractions have the same denominator you compute their difference by subtracting the numerators. If the fractions have unlike denominators first rewrite the fractions over a common denominator. Then compute the difference by subtracting the numerators, putting the answer over the common denominator, and reducing to lowest terms.

For example,

$$\frac{2}{3} - \frac{1}{5} = \frac{10}{15} - \frac{3}{15} = \frac{7}{15}$$

Models can again help to understand the subtraction of fractions. See page 369.

As before, we can recast a fraction subtraction problem as a fraction addition problem with a missing addend model:

Theorem. $\frac{a}{b} - \frac{c}{d} = \frac{e}{f}$ if and only if $\frac{a}{b} = \frac{e}{f} + \frac{c}{d}$.

This means that to solve the problem $a/b - c/d$ you could find what fraction you add to c/d to get back to a/b . To see how this model works, try some examples in one or more of the fraction models.

Proper Fractions and Mixed Numbers When doing arithmetic with fractions it is often helpful to find the simplest possible express fractions.

Definition. Consider the fraction $\frac{a}{b}$. If $a < b$ then $\frac{a}{b}$ is called a **proper fraction**. If $a \geq b$ then $\frac{a}{b}$ is called an **improper fraction**.

The fraction $\frac{2}{3}$ is proper while the fractions $\frac{3}{2}$ and $\frac{3}{3}$ are improper. If the fraction $\frac{a}{b}$ is improper b divides into a one or more times. So we can think of $\frac{a}{b}$ as a whole number and a fraction:

Definition. The sum of a natural number and a positive fraction is called a *mixed number*.

For example $2 + \frac{1}{3}$ is a mixed number. When writing a mixed number we will usually drop the plus sign and just write the whole part and the fractional part side by side:

$$2 + \frac{1}{3} = 2\frac{1}{3}.$$

To convert the mixed $A\frac{a}{b}$ to an improper fraction, multiply the whole number part A by the denominator b and add the result to the numerator:

$$A\frac{a}{b} = \frac{A \cdot b + a}{b}.$$

To convert the improper fraction $\frac{a}{b}$ to a mixed number, divide b into a . If $a \div b = qRr$, then the mixed number expression for $\frac{a}{b}$ is $q\frac{r}{b}$.

The easy way to add or subtract mixed numbers is to add the whole number parts and the fraction parts separately and then combine the answer as a new mixed number:

$$A\frac{a}{b} + B\frac{c}{d} = (A + B) \left(\frac{a}{b} + \frac{c}{d} \right).$$