Elementary Matrices and Frame Sequences

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Elementary Matrices

Definition. An elementary matrix E is the result of applying a combination, multiply or swap rule to the identity matrix.

An elementary matrix is then the **second frame** after a `combo`, `swap` or `mult` toolkit operation which has been applied to a **first frame** equal to the identity matrix.

Example:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

First frame = identity matrix.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{pmatrix}$$

Second frame
Elementary combo matrix
`combo(1,3,-5)`

Computer algebra systems and elementary matrices

The computer algebra system `maple` displays typical 4×4 elementary matrices (C=Combination, M=Multiply, S=Swap) as follows.

`with(linalg):`	`with(LinearAlgebra):`
`Id:=diag(1,1,1,1);`	`Id:=IdentityMatrix(4);`
`C:=addrow(Id,2,3,c);`	`C:=RowOperation(Id,[3,2],c);`
`M:=mulrow(Id,3,m);`	`M:=RowOperation(Id,3,m);`
`S:=swaprow(Id,1,4);`	`S:=RowOperation(Id,[4,1]);`

The answers:

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & c & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$S = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$